3rd year physics: Symmetry and Special Relativity lecture plan

A. M. Steane, October 5, 2010

Part A. *Lorentz transformation, kinematics and dynamics*

1. Lorentz transformation (stated not derived); how to define it in general; 4-vectors; invariant "length" of a 4-vector and the scalar product; proper time along a worldline, $dt/d\tau = \gamma$. Basic 4-vectors (position, velocity, acceleration, energy-momentum).

2. Rapidity, Doppler effect (e.g. transverse effect in atomic clock), headlight effect, transformation of velocity, examples (jets in particle physics and astrophysics), visual appearances ("superluminal" motion in astrophysical observations)

3. Force, rate of doing work, case of constant rest mass; transformation of force. Simple motion problems: constant speed $\rightarrow$ circular, constant force $\rightarrow$ hyperbolic.

4. Introduction to momentum conservation (Lewis and Tolman argument for a simple collision). Conservation of energy-momentum, relationship between the energy and momentum parts.

5. Collisions: in-flight decay, stationary target, CM frame, elastic, Compton (with worked examples).

6. Introduce the 4-gradient; continuity equation and J, wave equation, wave vector. Distortion of accelerated object exhibited by a simple example.

7. An aeroplane flying around a polygon; reference frames with orthogonal velocities, leading to a rotation (a simple example of Thomas precession); the product of two Lorentz transformations.

Part B. *Electromagnetism via 4-vectors*

8. Lorentz force equation; moving capacitor, transformation of the fields (motivated but stated without proof at this stage); moving current-carrying wire; field of uniformly moving point charge.

9. Vector potential; insensitivity of fields to some types of change in the potential; idea of a gauge condition; concise statement of Maxwell's equations using A.

10. The scalar and vector potential of a uniformly moving charge; solving the wave equation for a given source, by a sum of spherical wave solutions; the (retarded) potential due to an accelerating charge.

11. Fields of an accelerated charge exhibited (method of derivation understood but not

reproduced); dipole radiation for $v \ll c$; radiated power (Larmor); the half-wave antenna; synchrotron radiation.

12. The general idea of 3-dimensional tensors such as conductivity; the form $\mathbb{F}' = \Lambda \mathbb{F} \Lambda^T$ for the transformation of a matrix-like quantity; polar and axial vectors; introduction to index notation.

13. Tips for manipulation of relativistic equations; how to extract invariant scalar properties; relating an antisymmetric tensor to a pair of 3-vectors; the simplest way to obtain the transformation of electric and magnetic fields. The product rule for differentiation; the 4-curl.

14. Angular momentum, a 4-vector for intrinsic angular momentum (spin), helicity.

15. Reminder of Lagrangian mechanics; least action and most proper time; Lagrangian for electromagnetic interaction; Hamiltonian for a particle moving in a magnetic field. The connection between symmetry and conservation laws.

16. The Maxwell equations and the Lorentz force equation expressed using a matrix combining $\mathbf{E}$ and $\mathbf{B}$; why it is useful; invariants ($c^2B^2 - E^2$ and $\mathbf{B} \cdot \mathbf{E}$). Parity inversion symmetry.

17. Brief survey of static field case (existence of a frame with zero $\mathbf{E}$ or $\mathbf{B}$). Poynting's argument and Poynting vector.

18. Momentum flow, stress-energy tensor, conservation of energy-momentum (examples: waves, capacitor, solenoid).

Part C. *The simplest classical fields (generalizations of the wave equation)*

19. Rotations of a 2-component complex vector ('spinor'); extension to Lorentz transformation; significance of parity inversion.

20. 2 types of spinor (right and left handed), Dirac spinor, Lorentz transformation of Dirac spinor leading to a classical (not quantum) version of Dirac's equation; how it gives the energy-momentum vector and the spin 4-vector.

21. The wave equation interpreted in terms of energy and momentum; the introduction of a mass term to give Klein-Gordan equation; equations for associated currents and conserved quantities, plane wave solutions (brief survey). Extracting first-order equations: Weyl and Dirac (now a wave equation but still classical).

22. Properties of the Dirac equation; current, conserved quantities; plane wave solutions. Mention the basic idea of quantum theory: to associate an oscillator with each mode, and interpret solutions as particle/antiparticle.

23. The final lecture will draw things together, summarizing what have been the essential concepts and dominant themes of the course, and briefly introducing some wider questions, such as the union of quantum theory and relativity. There will also be a question and answer session.

1 Textbooks

The material on basic relativity (Doppler, transformation of velocity, etc.) is standard and can be found in many basic textbooks. However, I have devoted some time to finding ways of making the derivations as straightforward as possible.

The material on electromagnetism overlaps with textbooks such as Bleaney and Bleaney, Griffiths, Lorraine and Corson, and the Feynman lectures. However it begins the journey beyond them. I have found books by Rindler and Jackson helpful in preparing the course, but both are too technical and concise for most students at this level. The lecture notes will, I hope, be sufficient to allow students to profit from such further reading as they choose.

The treatment of spinors seems to be rather confused in the textbooks at present. They are introduced in quantum mechanics books but not in classical mechanics books, giving a false impression of their general nature. A helpful rule of thumb is the following: the quantum and classical treatments have much in common, they differ only when it comes to measurements of the physical quantities represented by a given spinor. Classically, we extract 3- and 4-vectors which are completely measurable; in quantum theory there is the unavoidable disturbance associated with measurement.

The classical wave equations associated with Klein-Gordan and Dirac are introduced in books on the classical theory of fields, but the ones I know of are a bit too concise. Therefore the lecture notes will cover what is needed. However it is useful to have more than one resource. Introductory accounts at the right level may be found in introductory particle physics or quantum mechanics books, but there is again a confusing tendency to imply that these equations describe the evolution of a single-particle wavefunction (i.e. to repeat the mistake made by early workers, only to abandon it later in the book). As long as the student regards the function in question as a classical scalar field or spinor field, the textbook accounts of currents and conserved quantities are useful.