Some basic points in thermodynamics

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August 16, 2022

1 Always true

$$\Delta U = \Delta Q + \Delta W + \Delta(\text{chemical energy}) \tag{1}$$
$$= \Delta Q + \Delta W \qquad \text{for closed system} \tag{2}$$

The chemical energy part is the energy associated with material moving into or out of the system, which does not happen for a closed system.

$$\mathrm{d}U = T\mathrm{d}S - p\mathrm{d}V + \mu\mathrm{d}N \qquad \text{for } pV \text{ system} \tag{3}$$
$$= T\mathrm{d}S - p\mathrm{d}V \qquad \text{for closed } pV \text{ system} \tag{4}$$

For closed system:

$$C_v \equiv \frac{đQ_V}{dT} = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V, \tag{5}$$
$$C_p \equiv \frac{đQ_p}{dT} = T\left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p \tag{6}$$

$$\gamma \equiv \frac{C_p}{C_v} \tag{7}$$

2 Ideal gas

Definition: Boyle's law and $U = U(T)$.
In consequence:

$$pV = nRT, \qquad \Delta U = \int C_V \mathrm{d}T \tag{8}$$

So

$$C_p = C_V + p\left(\frac{\partial V}{\partial T}\right)_p = C_V + nR \tag{9}$$

and therefore

$$\gamma = 1 + \frac{nR}{C_V} \tag{10}$$

It is often assumed, but it is not necessarily true (for an ideal gas), that C_V is independent of temperature. If it is, then clearly so is C_p and γ. In this case, pV^γ is constant for an adiabatic process.

For a monatomic gas, one finds to very good approximation $C_V = (3/2)nR$ so then $\gamma = 5/3$.