CALCULUS ANSWERS
Courtesy of Prof. Julia Yeomans

A1(a) (i) $(\cos x + 3x^2 \sin x)e^{x^3}$ (ii) $(x^3 \cos x + 3x^2 \sin x)e^{x^3 \sin x}$ (iii) $\frac{-\tanh(1/x)}{x^2}$

(b) (i) $-\frac{1}{\sqrt{1-x^2}}$ (ii) $\frac{1}{(1+2x)}$

(c) (i) $x^{\cos x}(\frac{\cos x}{x} - \sin x \ln x)$ (ii) $\frac{2}{x \ln 10}$

(d) (i) $\frac{2x-(y^2/x)e^{y \ln x}}{(ylnx+1)e^{ylnx}-2y}$

(e) $\frac{dy}{dx} = \coth\theta$; $\frac{d^2y}{dx^2} = -\mathrm{cosech}^3\theta$

A6 $\frac{d^8y}{dx^8} = x^2 \sin x - 16x \cos -56 \sin x$

A7 Solution in Riley, Hobson & Bence problem 2.14

B1(a) (i) $\frac{-1}{(1+2ax+x^2)^{1/2}} + C$ (ii) $e - 1$ (iii) $2/3$ (iv) 4

(b) (i) $\sin^{-1}(\frac{x-1}{2}) + C$ (ii) $\pi/4$

(c) (i) $\ln(\frac{x}{\sqrt{1+x^2}}) + C$

(d) (i) $\sin x - x \cos x + C$ (ii) $x \ln x - x + C$

(e) 1

(f) (i) $\frac{1}{5}\sin^5 x - \frac{1}{3}\sin^3 x + C$ (ii) $-\frac{1}{5}\cos^5 x + \frac{2}{7}\cos^7 x - \frac{1}{9}\cos^8 x + C$ (iii) $\frac{x}{16} - \frac{1}{32}\sin(2x) + \frac{1}{48}\sin^3(2x) + C$

(g) (i) $(x^2-9)^{\frac{1}{2}} - 3\arctan((x^2-9)^{\frac{1}{2}}/3) + C$ (ii) $-\frac{(16-x^2)^{\frac{1}{2}}}{16x} + C$

B3 (a) $\sinh 1$ (b) $\pi/2$ (c) $S.A. = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$

B4 (a) $32/3$ (b) $64/3$ (c) (i) 1 (ii) 1

C2 $(a)(i)$ $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ (ii) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - ...$ (iii) $x - \frac{x^3}{3} + ...$

(b) 0.515038072; error in last digit

C4 0.9461

C6(a) (i) 1 (ii) 1 (iii) 0

(c) (i) 0 (ii) 0 (iii) -1

(d) (i) maximum (ii) ∞

D2(a) (i) $\frac{x}{(x^2+y^2)^{1/2}}$ $\quad (ii)$ $-\frac{y}{x^2+y^2}$ $\quad (iii)$ $y^x \ln y$

D3 9%

D4 (a) $na\cos^{n-1} at \sin^{n-1} at(\cos^2 at - \sin^2 at)$ $\quad (b)$ $x + 2x\ln x - \frac{1}{x(\ln x)^2}$

D5 $\frac{\partial w}{\partial r} = -2(x^2+y^2)^{\frac{1}{2}} e^{-x^2-y^2}$, $\quad \frac{\partial w}{\partial \theta} = 0$

D7 (a) $(\frac{\partial z}{\partial x})_y = (\frac{\partial z}{\partial u})_v 2x + (\frac{\partial z}{\partial v})_u 2y$

(b) $(\frac{\partial z}{\partial u})_v = \frac{1}{2(x^2-y^2)}\{x(\frac{\partial z}{\partial x})_y - y(\frac{\partial z}{\partial y})_x\}$

(c) $(\frac{\partial z}{\partial u})_v - (\frac{\partial z}{\partial v})_u = \frac{1}{2(x-y)}\{(\frac{\partial z}{\partial x})_y - (\frac{\partial z}{\partial y})_x\}$

D8 $f(x,y) = e^6 + 3e^6(x-2) + 2e^6(y-3) + \frac{9e}{2}^6(x-2)^2 + 6e^6(x-2)(y-3) + 2e^6(y-3)^2$

D9 (i) $x = y = 0$ (minimum)

(ii) $x = y = 0$ (maximum); $\quad x = y = 1/3$ (saddle)

(iii) $x = y = \pi/3$ (maximum)

D10 (i) exact $f = xy$

(ii) not exact

(iii) exact $f = \frac{1}{2}(x^2+y^2+z^2)$

(b) 0

E5 (i) $-\pi$ (ii) oscillates (iii) $\sqrt{2}/4$ (iv) 0 (v) 1

E6 $y(x) = -3x + 4x^3$