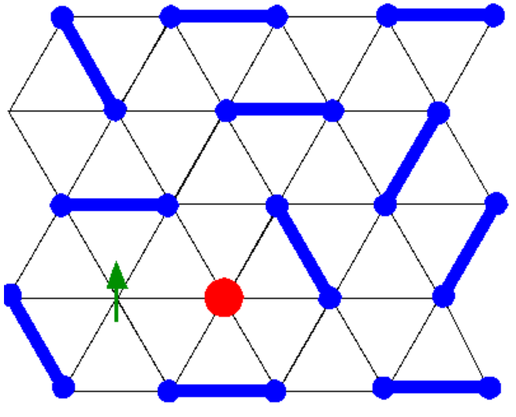
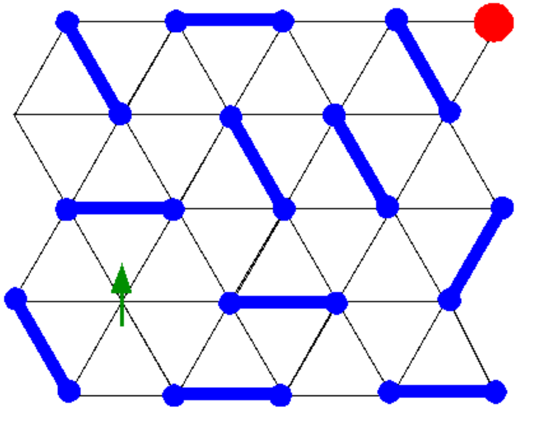
Fractionalisation in lattice models: from high-T_c to magnetic monopoles



Roderich Moessner

MPI-PKS Dresden



Some general themes

control and manipulation of d.o.f. and interactions

$$\Longleftrightarrow$$

progress in many-body physics

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Experiment: new physical phenomena

Some general themes

control and manipulation of d.o.f. and interactions

$\Leftrightarrow$

progress in many-body physics

Experiment: new physical phenomena

Models

- standard (simple)
  - Hubbard model
  - Ising model
- custom-tailored
  - Rokhsar Kivelson
- soluble
  - low-dimensional

Phases and transitions

- fractionalised phases
- ???

Dynamics

- real-time quantum
- out of equilibrium

Disorder

Fractionalisation

High- and low-energy descriptions often have little in common

- molecules $\longrightarrow$ waves
- band electrons $\longrightarrow$ spin waves

Fractionalisation: ‘quantum numbers’ not simply related

- spin-charge separation (high-T$_c$?)
- Laughlin quasiparticles (charge $e/3$)
- magnetic monopoles (spin ice)

Cold atom realisations of fractionalised phases

- Kitaev II model (non-Abelian anyons)
  - Quantum mechanical toolbox Zoller, Buchler, ...
- Triangular lattice topological phases
  - four-spin plaquette exchange Misguich, Lhuillier
  - tunable n.n. exchange
  - Klein models
  - quantum dimer models
- classical fractionalisation in $d = 3$
  - simple Ising model with non-collinear axes
  - nearest-neighbour or long-range dipoles do the trick!

Fluctuations and quantum dimer models

Fluctuations (thermal, quantum, . . .) destroy order.

$\Rightarrow$ what happens instead?

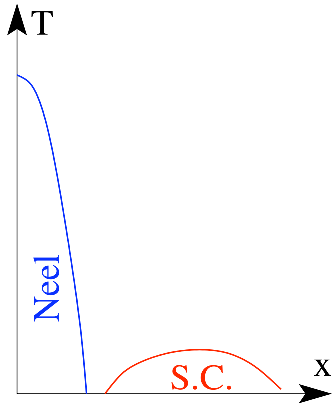
$\Rightarrow$ QDMs capture several aspects of new physics

Outline

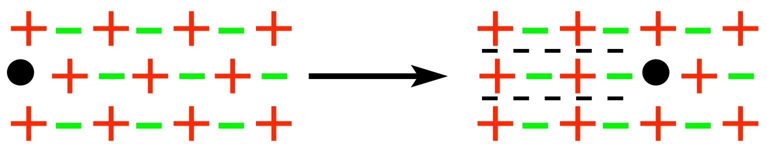
- historical perspective: high-temperature superconductors
  - spin liquids and fractionalisation
- quantum dimer models
  - phase diagram
  - liquidity and deconfinement
  - topological order
- Outlook

Background: short-range RVB physics

Basic problem of high-T$_c$: how do holes hop through an antiferromagnetic Mott insulator on square lattice?



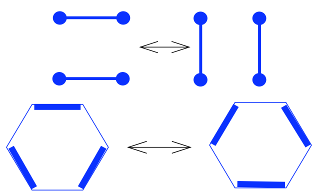
Hole motion is frustrated:
hopping creates domain walls



Possible resolution: magnet enters a different phase
resonating valence bond liquid phase
which breaks no symmetries. Neighbouring electrons form a singlet ("valence") bond, denoted by a dimer: $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \sim$ 

The basic RVB scenario - electron fractionalisation

Energetics	RVB	Neel
single pair	valence bond optimal	
higher coordination	energy from resonance	...each neighbour
hole doping	motion unimpeded	motion frustrated



- Basic resonance move is that of benzene

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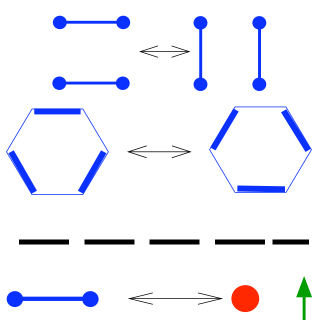
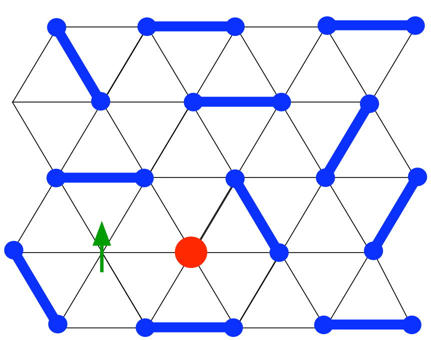
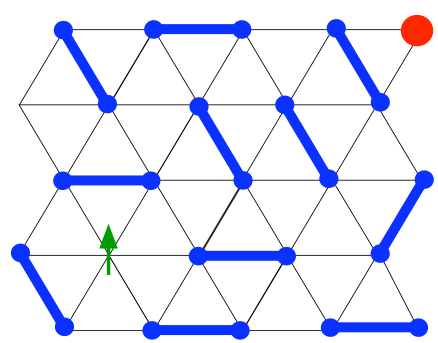


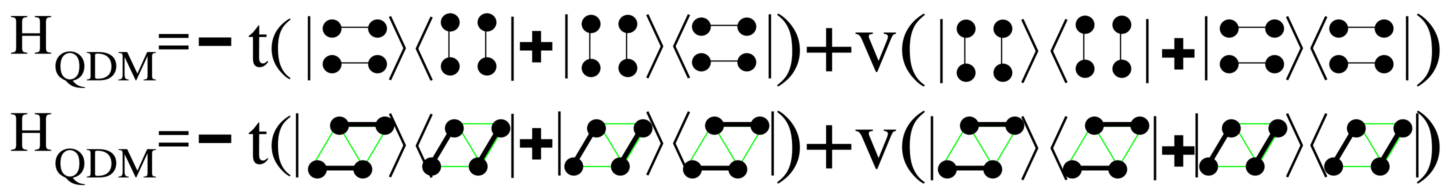
- Basic resonance move is that of benzene
- Removing an electron $\longrightarrow$ holon + spinon





spinon and holon are deconfined
$\downarrow$
(bosonic) holons can condense

The Rokhsar-Kivelson quantum dimer model



- Hilbert space: exponentially numerous dimer coverings
- Resonance (t) and potential (v) term from uncontrolled approximation – one parameter: v/t
- RK point $v/t = 1$ is exactly soluble in $d = 2$ at $T = 0$:

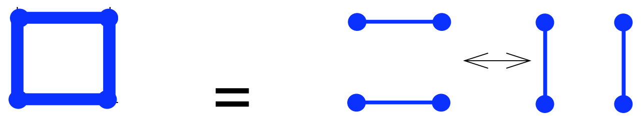
$$|0\rangle = \frac{1}{\sqrt{N_c}} \sum_c |c\rangle \rightarrow \langle \hat{P} \rangle = \frac{1}{N_c} \sum_{c,c'} \langle c|\hat{P}|c'\rangle = \frac{1}{N_c} \sum_c p_c$$

$\longrightarrow$ classical calculation for diagonal operators

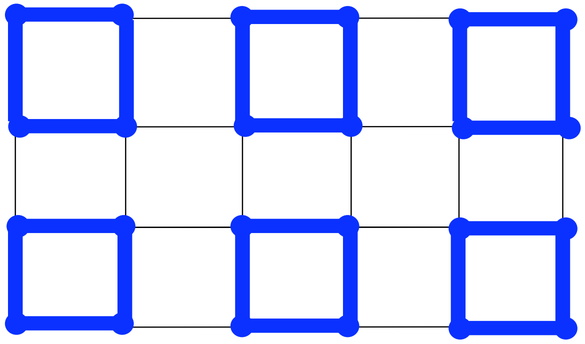
- $v/t > 1$ and limits of $v/t \rightarrow -\infty$ give solid (staggered and columnar, respectively) phases:

The enemy: order by disorder

- Consider $v = 0$: only term in H_{QDM} is kinetic term
- kinetic term gains energy from resonating plaquette:

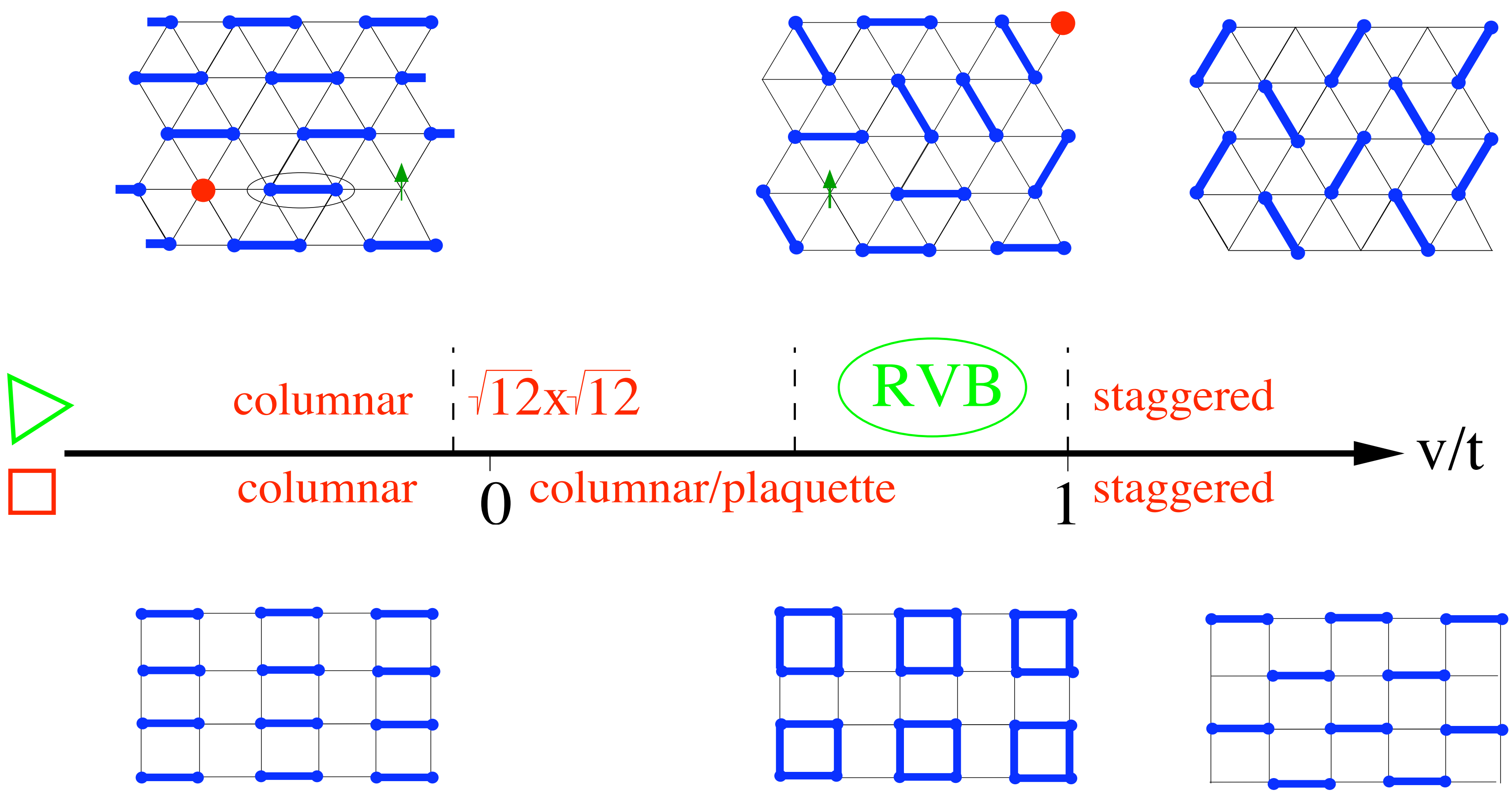


- Maximal energy gain $\longrightarrow$ dense packing
- Dense packing $\longrightarrow$ crystallinity
- Crystallinity $\longrightarrow$ symmetry breaking: ‘order by disorder’



- Plaquette solid: only variational guess!

Phase diagram for square and triangular lattices

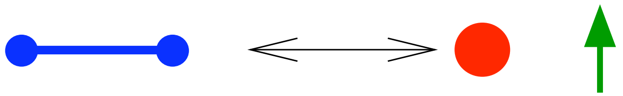


All phases on square lattice are confining RK; Sachdev; ...

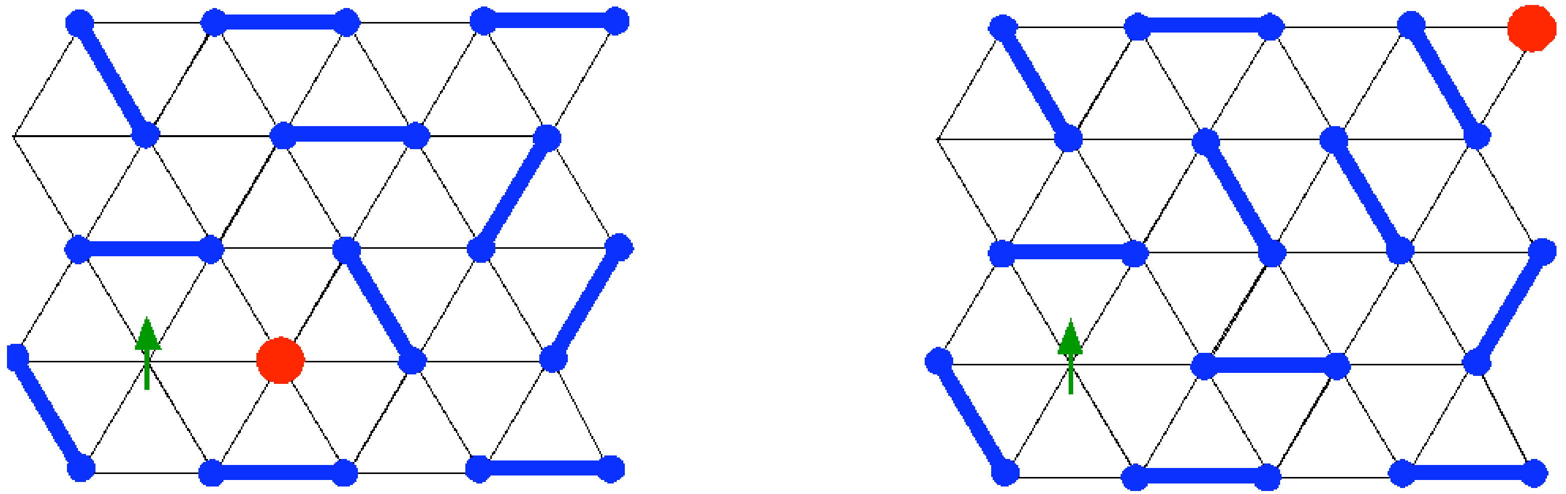
Triangular lattice has *bona fide* RVB phase

Liquidity and fractionalisation

- Removing an electron: holon (S=0) and spinon (q=0)

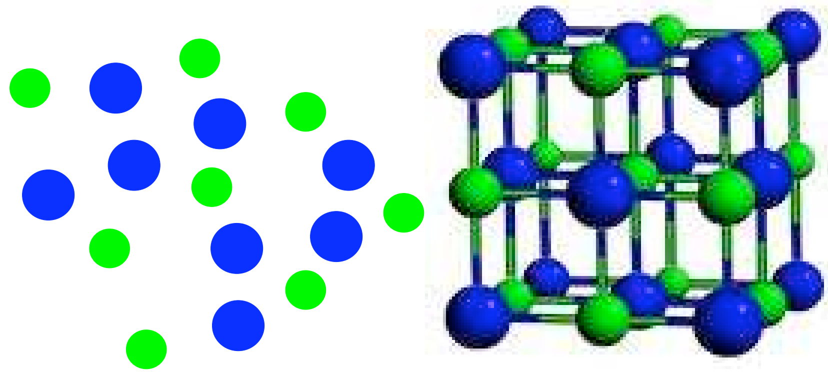


- Spinon and holons are deconfined: spin-charge separation

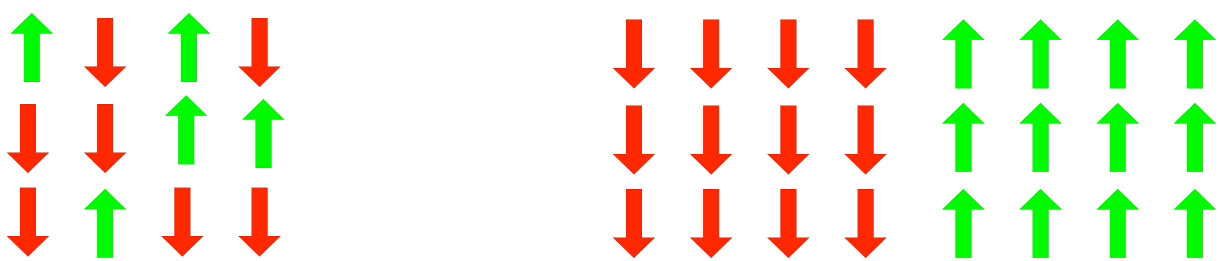


Anything beyond conventional order and disorder?

Gas-crystal (e.g. rock salt):



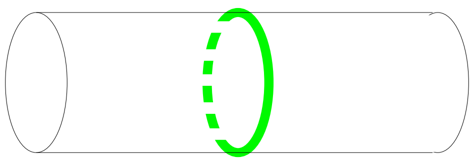
Paramagnet-ferromagnet



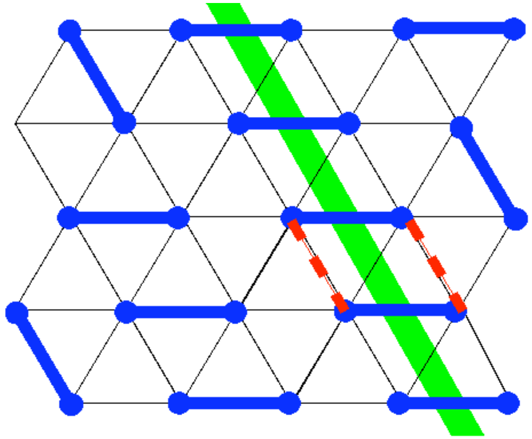
Anything else???

Liquidity and topological order

Topological order on surface of non-trivial topology (e.g. cylinder)



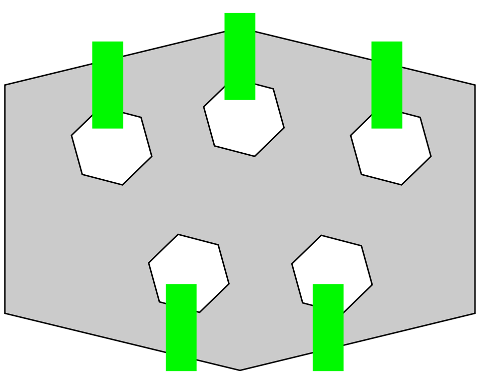
- Winding parity $\mathcal{P}$ with respect to cut is invariant under action of H_{RK} $\Rightarrow$ $\mathcal{P}$ labels topological sectors
- Liquids locally indistinguishable $\Rightarrow$ ground states $|e\rangle, |o\rangle$ degenerate for $L \to \infty$:
  'topological degeneracy/order' Wen
- Unlike conventional order: degeneracy due to breaking of local symmetry

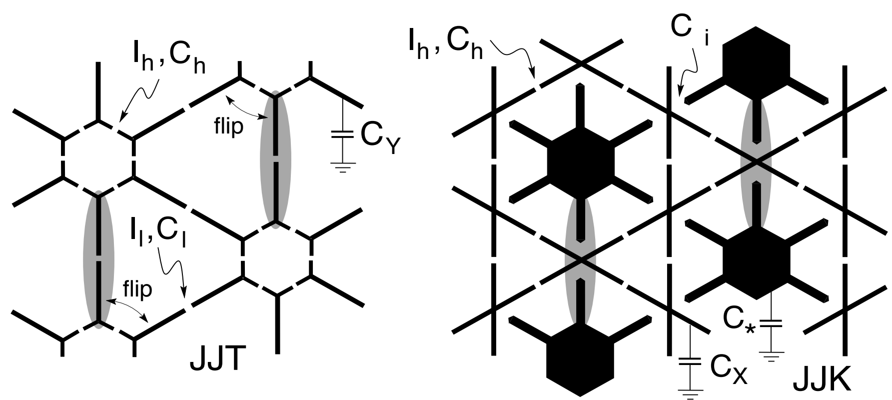


Topological quantum computing *Kitaev; Ioffe* et al.

Topological protection: Use $|\mathcal{P}\rangle = |e\rangle, |o\rangle$ as q-bit Kitaev

- Liquids locally indistinguishable: $E_N^e - E_N^o \propto \exp(-L)$
  $\Rightarrow$ local noise H_N <u>cannot</u> lead to dephasing
- Proposal is scalable: many cuts in single chip
- Implementation as Josephson-junction array Ioffe *et al.*
- Problem: logic gates; non-local operations, ...





High-dimensional fractionalisation

Fractionalisation through frustration

- frustrated Ising models
  - ground-state degeneracy
  - spin ice
- equivalence of short- and long-range interactions
- topological phase with emergent quasiparticles
  - monopoles and artificial photons
  - algebraic correlations without criticality
- dimensional reduction

Conventional vs frustrated Ising models

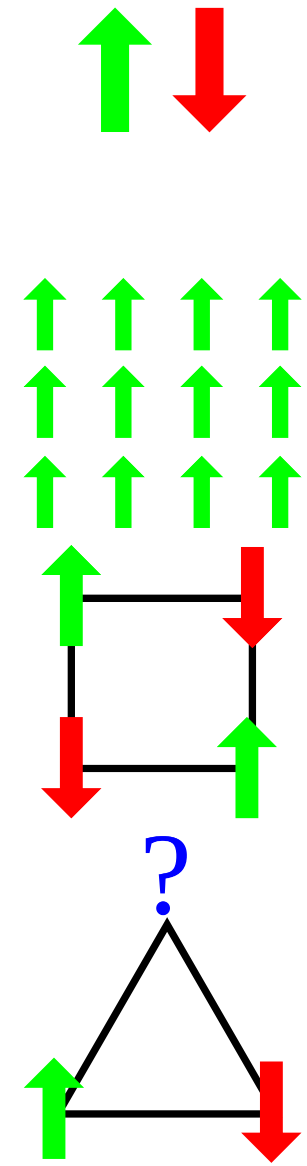
- Consider classical Ising spins, pointing either up or down: $\sigma_i = \pm 1$

- Simple exchange (strength J):

$$\mathcal{H} = J\sigma_i\sigma_j$$

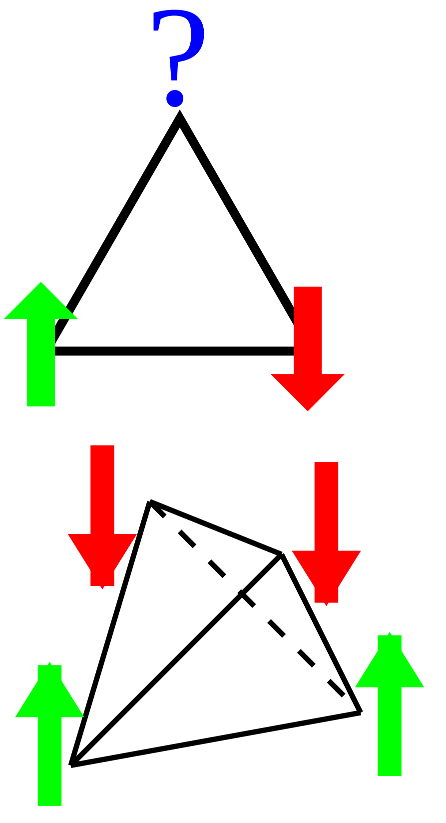
  - $J < 0$: ferromagnetic – spins align
  - $J > 0$: antiferromagnetic – spins antialign
  - . . . but only where possible: 'frustration'

  $\Longrightarrow$ What happens instead?



Frustration leads to (classical) degeneracy

Not all terms in $\mathcal{H} = \sum_{\langle ij \rangle} \sigma_i \sigma_j$ can simultaneously be minimised



- But we can rewrite $\mathcal{H}$:

$$\mathcal{H} = \frac{J}{2} \left(\sum_{i=1}^{q} \sigma_i \right)^2 + const$$

which can be minimised

- for tetrahedron: $\sum_i \sigma_i = 0$

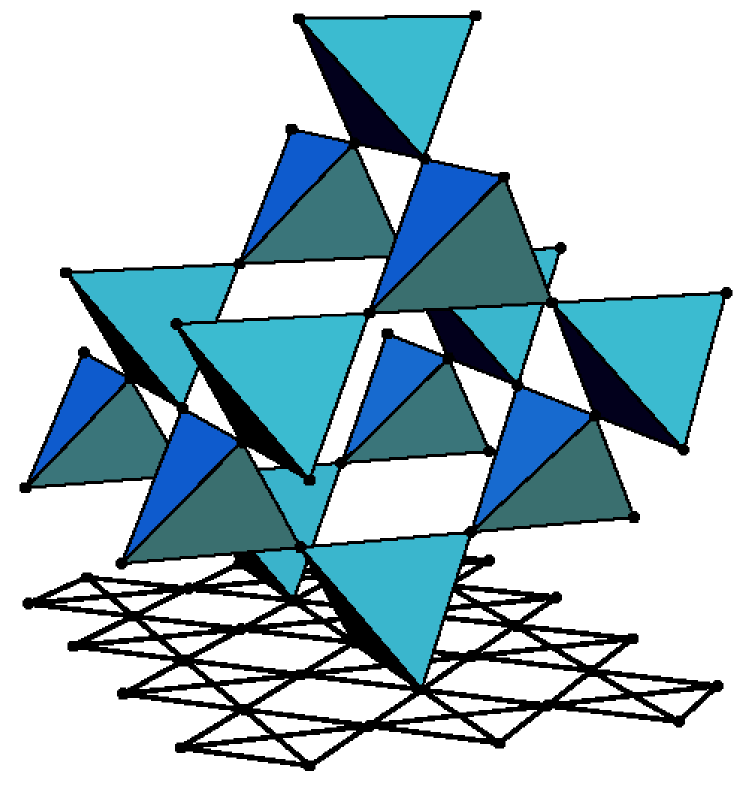
$\Rightarrow N_{gs} = \binom{4}{2} = 6$ ground states

Degeneracy is hallmark of frustration ($\Rightarrow$ quantum Hall!)

Zero-point entropy on the pyrochlore lattice

- Pyrochlore lattice = corner-sharing tetrahedra

$$\mathcal{H}_{pyro} = \frac{J}{2} \sum_{tet} \left(\sum_{i \in tet} \sigma_i \right)^2$$



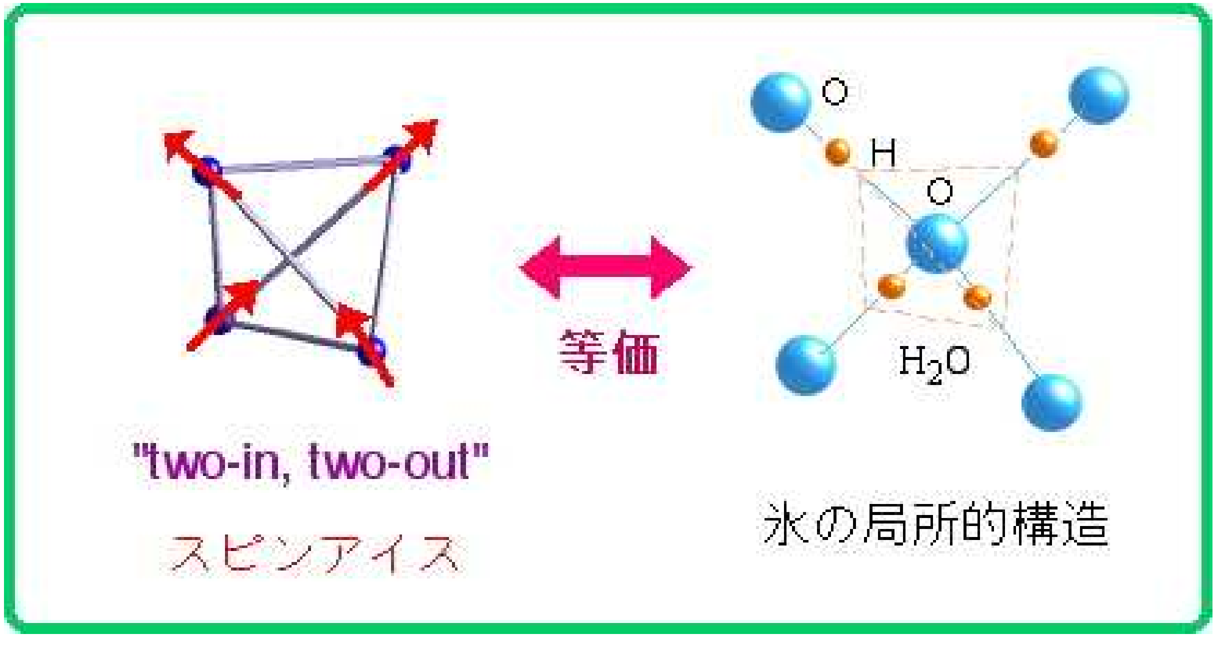
- Pauling estimate of ground state entropy $\mathcal{S}_0 = \ln N_{gs}$:

$$N_{gs} = 2^N \left(\frac{6}{16} \right)^{N/2} \Rightarrow \mathcal{S}_0 = \frac{1}{2} \ln \frac{3}{2}$$

- microstates vs. constraints; N spins, $N/2$ tetrahedra

Mapping from ice to spin ice

- In ice, water molecules retain their identity
- Hydrogen near oxygen ↔ spin pointing in



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- axes non-collinear!

A dipolar Hamiltonian of spin ice *Siddharthan+Shastry*

- Simple nearest-neighbour model:

$$\mathcal{H}_{nn} = -J \sum_{\langle ij \rangle} \vec{\mu}_i \vec{\mu}_j$$

- For polar molecules with dipole moment μ:

$$\mathcal{H} = \mathcal{H}_{nn} + \frac{\mu_0}{4\pi} \sum_{ij} \frac{\vec{\mu}_i \cdot \vec{\mu}_j - 3(\vec{\mu}_i \cdot \hat{r}_{ij})(\vec{\mu}_i \cdot \hat{r}_{ij})}{r_{ij}^3}$$

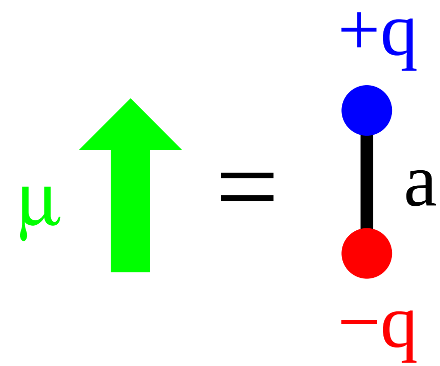
- Both give same entropy (!!!) Gingras *et al.*

WHY???

The 'dumbell' model

Dipole ≈ pair of opposite charges ($\mu = qa$):

- Sum over dipoles ≈ sum over charges:



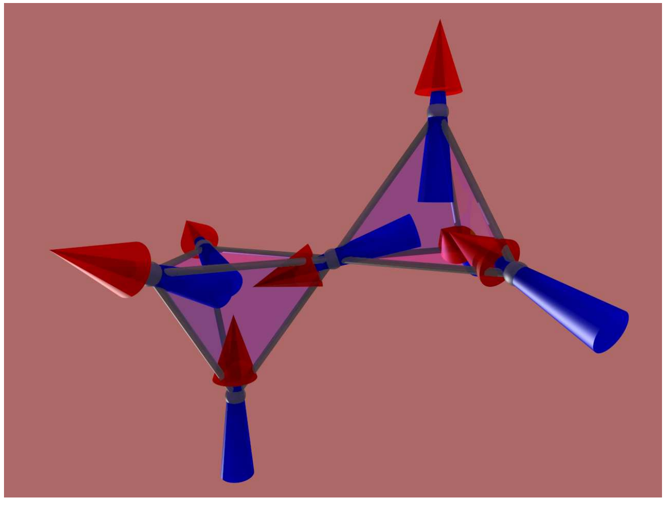
$$\mathcal{H}_{ij} = \sum_{m,n=1}^{2} v(r_{ij}^{mn})$$

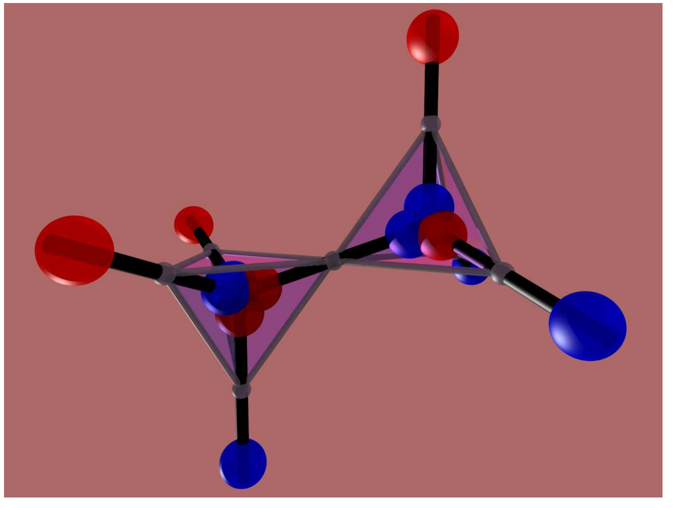
- $v \propto q^2/r$ is the usual Coulomb interaction (regularised):

$$v(r_{ij}^{mn}) = \begin{cases} \mu_0\, q_i^m q_j^n / (4\pi r_{ij}^{mn}) & i \neq j \\ v_o(\frac{\mu}{a})^2 = \frac{J}{3} + 4\frac{D}{3}(1 + \sqrt{\frac{2}{3}}) & i = j, \end{cases}$$

Origin of the ice rules

Choose $a = a_d$, separation between centres of tetrahedra





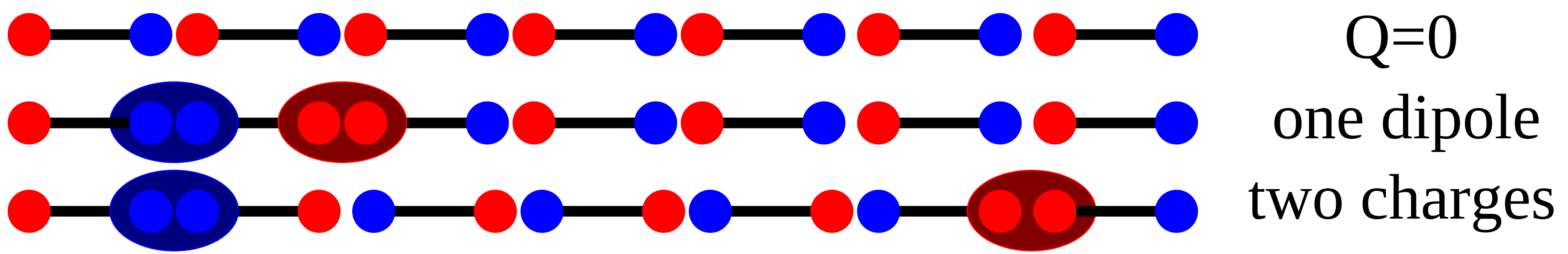
Resum tetrahedral charges $Q_\alpha = \sum_{r_i^m \in \alpha} q_i^m$:

$$\mathcal{H} \approx \sum_{ij}^{mn} v(r_{ij,mn}) \longrightarrow \sum_{\alpha\beta} V(r_{\alpha\beta}) = \begin{cases} \frac{\mu_0}{4\pi} \frac{Q_\alpha Q_\beta}{r_{\alpha\beta}} & \alpha \neq \beta \\ \frac{1}{2} v_o Q_\alpha^2 & \alpha = \beta \end{cases}$$

- Ice configurations ($Q_\alpha \equiv 0$) degenerate$\Rightarrow$ Pauling entropy!

Excitations: dipoles or charges?

- Ground-state
  - no net charge
- Excited states:
  - flipped spin $\leftrightarrow$ dipole excitation
  - same as two charges?



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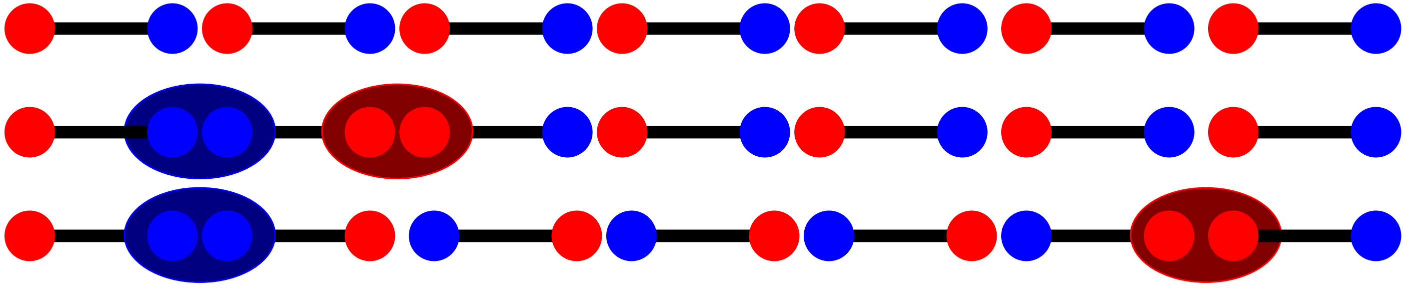


Q=0

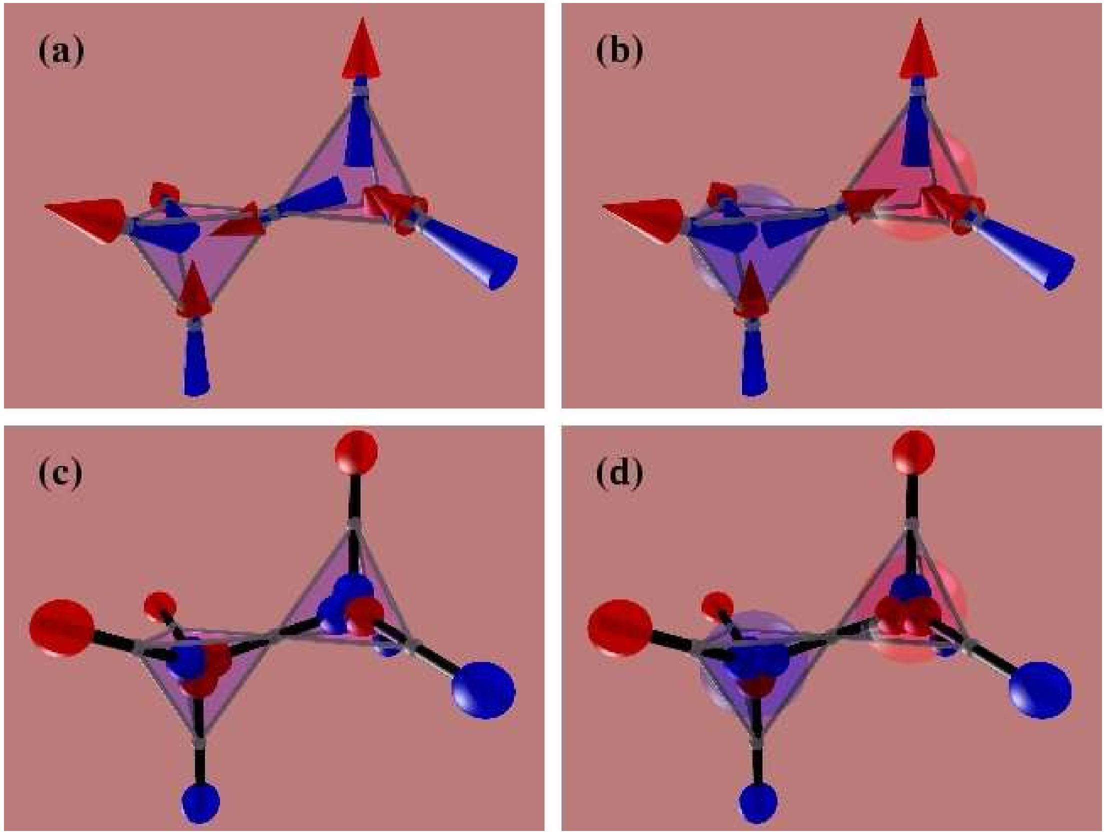
one dipole

two charges

Fractionalisation in $d = 1$

Excitations in spin ice: dipolar or charged?

Single spin-flip (dipole μ)
$\equiv$
two charged tetrahedra
(charges $q_m = 2\mu/a_d$)



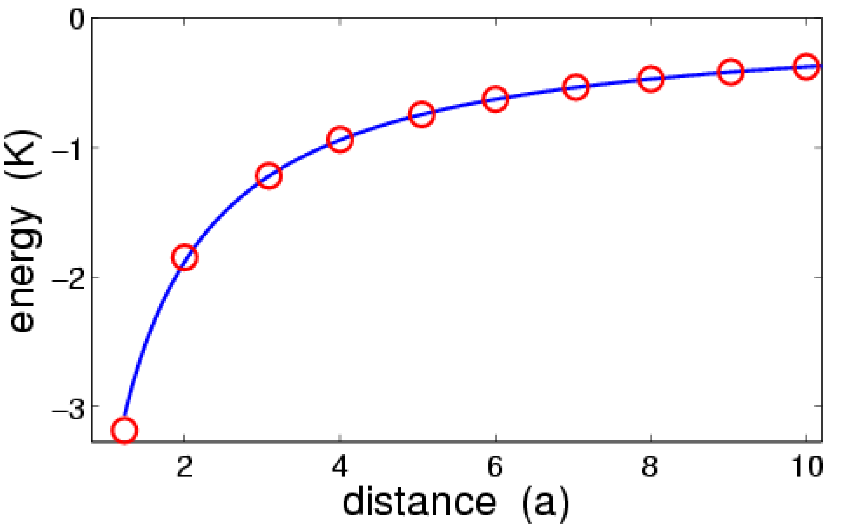
Are charges independent?
$\Rightarrow$ Fractionalisation in $d = 3$?

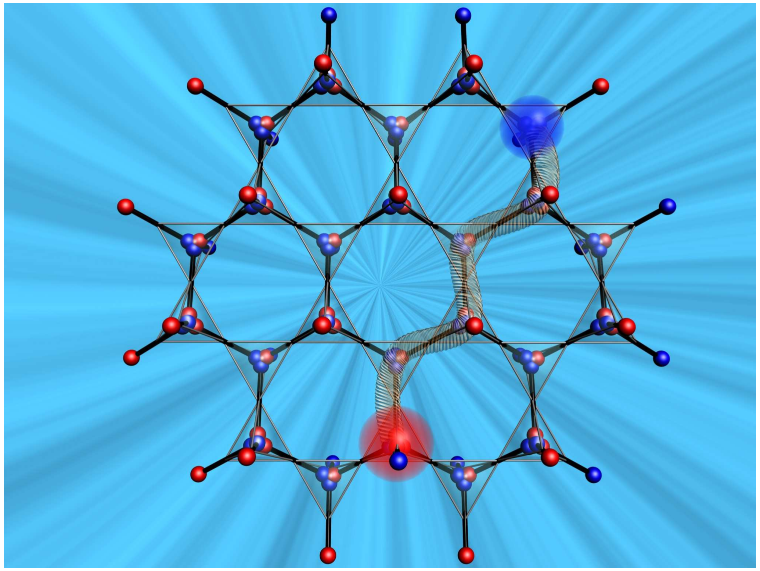
Deconfined monopoles

Dumbell Hamiltonian gives

$$E(r) = -\frac{\mu_0}{4\pi}\frac{q_m^2}{r}$$

- magnetic Coulomb interaction for magnetic dipoles
- electric Coulomb interaction for electric dipoles





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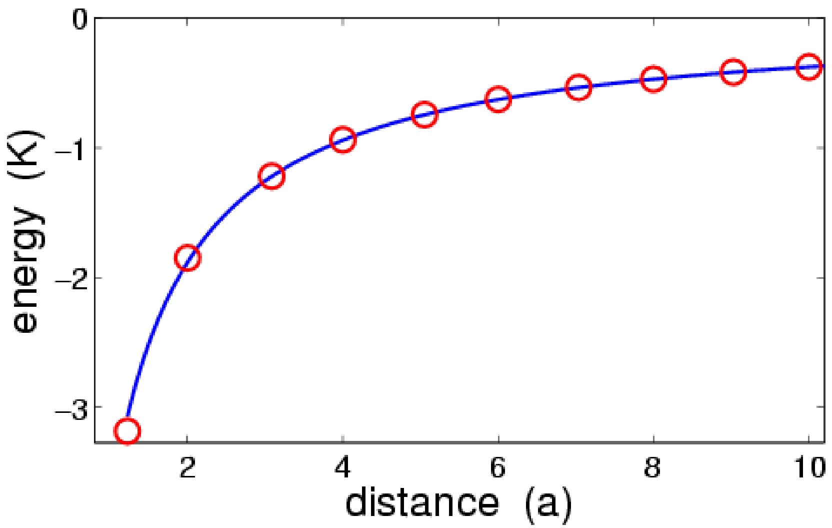
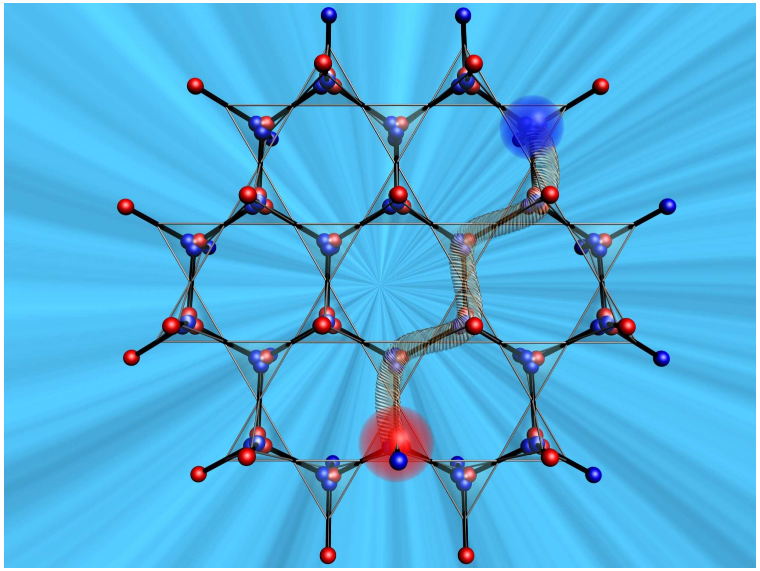


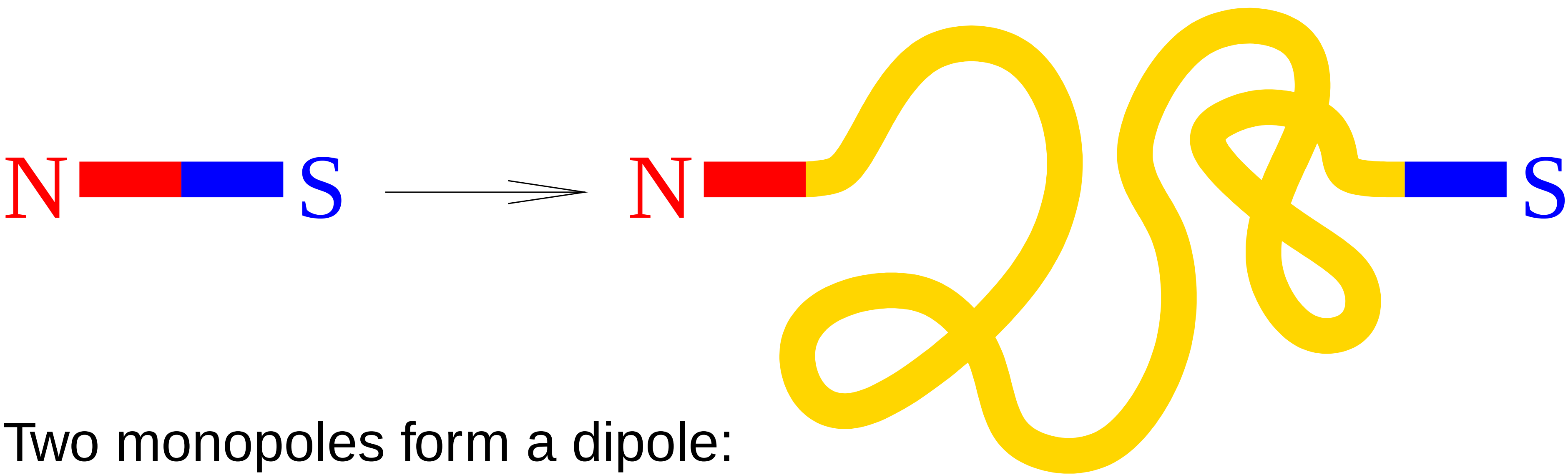


- magnetic Coulomb interaction for magnetic dipoles
- electric Coulomb interaction for electric dipoles
- deconfined monopoles (in $\vec{H}$)

Intuitive picture for monopoles

Simplest picture does not work: disconnect monopoles

Next best thing: no string tension between monopoles:



Two monopoles form a dipole:

- connected by tensionless ‘Dirac string’
- Dirac string <u>is</u> observable

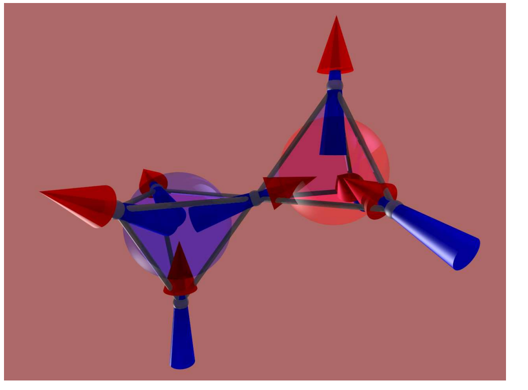
Kagome ice: dimensional reduction in a field

Ising axes are not collinear

- $[111]$ field pins one sublattice of spins

$\vec{B}$

$\Uparrow$



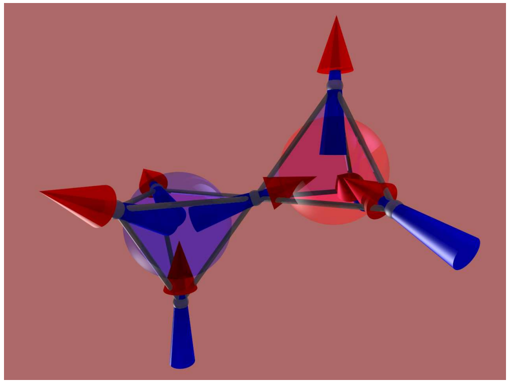
Kagome ice: dimensional reduction in a field

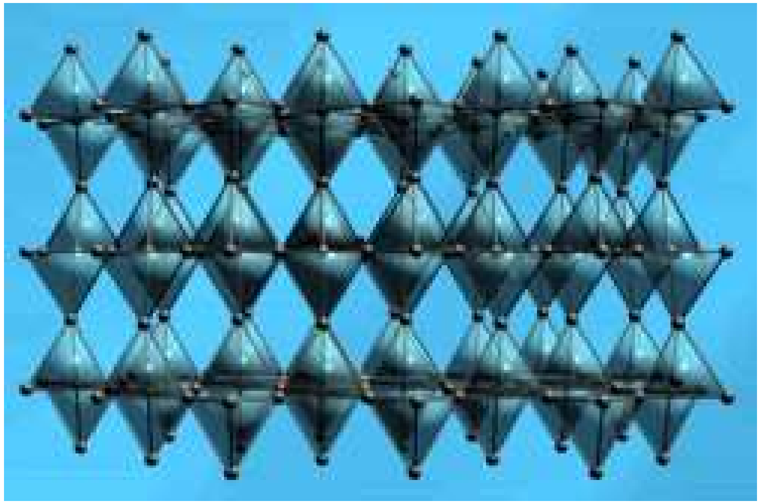
Ising axes are not collinear

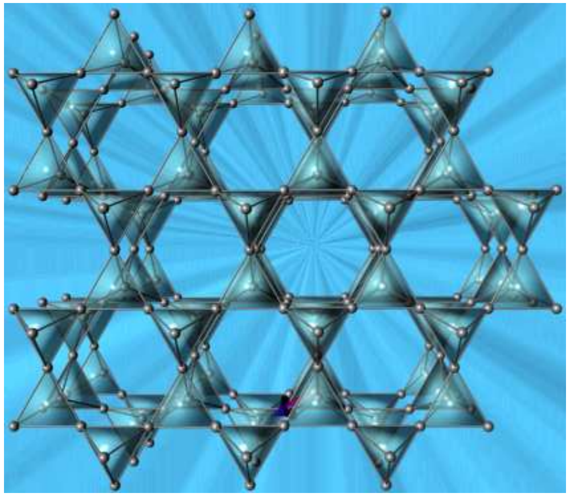
- $[111]$ field pins one sublattice of spins
- Other sublattices form kagome lattice
- Kagome lattice: two-dimensional
- Can change effective dimensionality without touching lattice

$\vec{B}$

$\Uparrow$

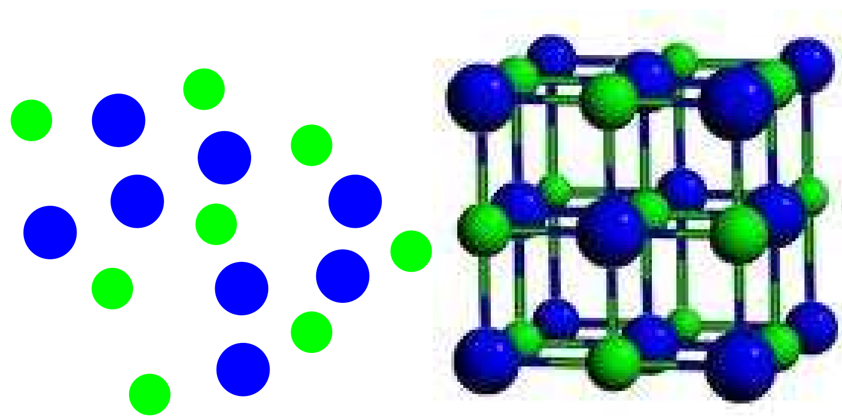




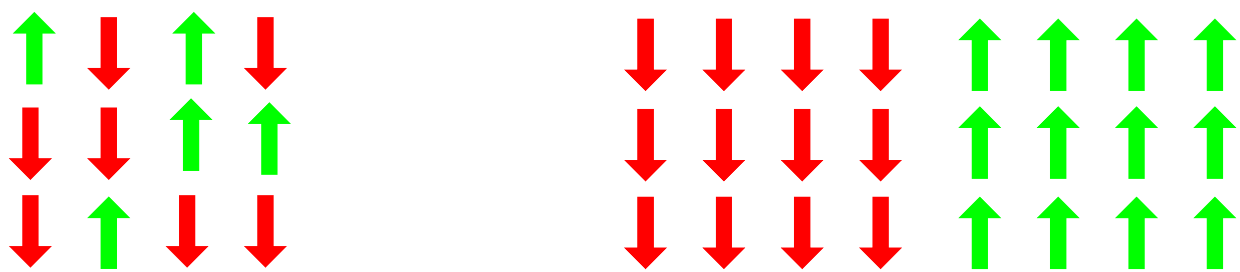


Conventional order and disorder

Gas-crystal (e.g. rock salt):



Paramagnet-ferromagnet (e.g. fridge magnet)



In between: critical points

Anything else???

Is spin ice ordered or not?

No order as in ferromagnet

- deconfined monopoles

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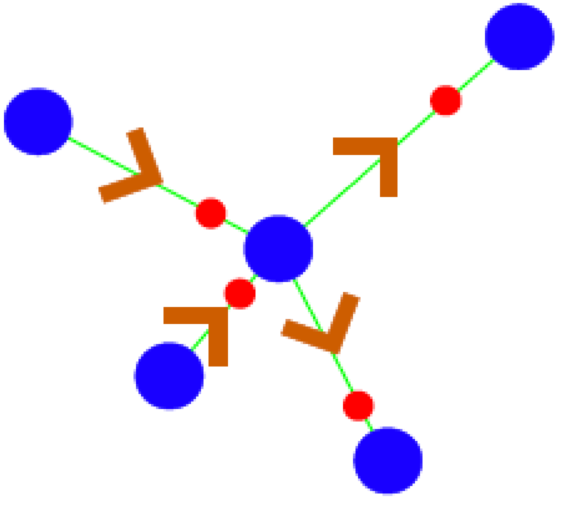
Not disordered like a paramagnet

- ice rules

Is spin ice ordered or not?



No order as in ferromagnet

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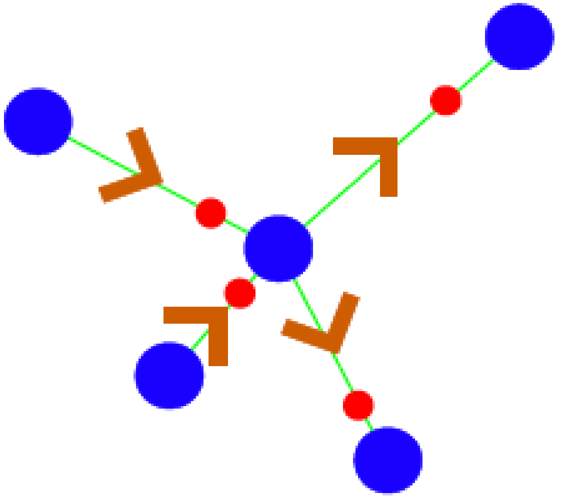
Not disordered like a paramagnet

- ice rules $\Rightarrow$ 'conservation law'

Consider magnetic moments $\vec{\mu}_i$ as (lattice) 'flux' vector field

- Ice rules $\Leftrightarrow \nabla \cdot \vec{\mu} = 0 \Longrightarrow \vec{\mu} = \nabla \times \vec{A}$

Is spin ice ordered or not?



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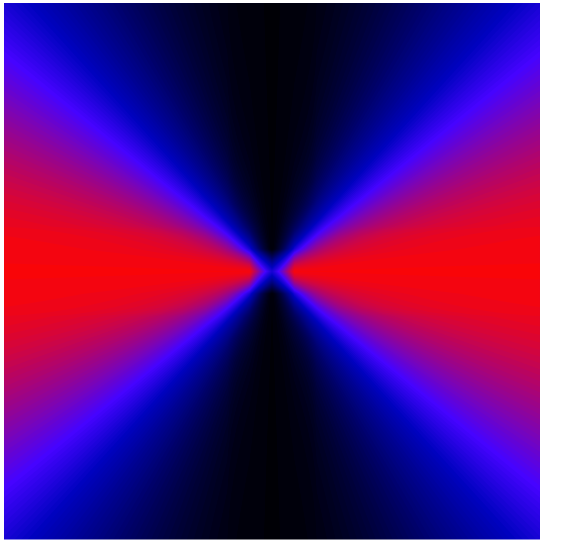
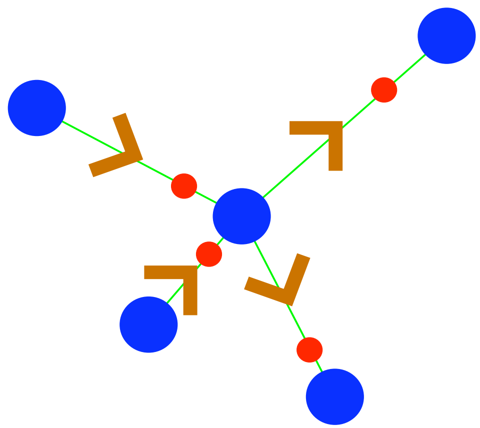


- Ice rules $\Leftrightarrow \nabla \cdot \vec{\mu} = 0 \Longrightarrow \vec{\mu} = \nabla \times \vec{A}$
- Local constraint
  $\Rightarrow$ ‘emergent gauge structure’
- Bow-tie motif in neutron scattering
- Algebraic (but not critical!) correlations

Spin ice correlations: emergent gauge structure

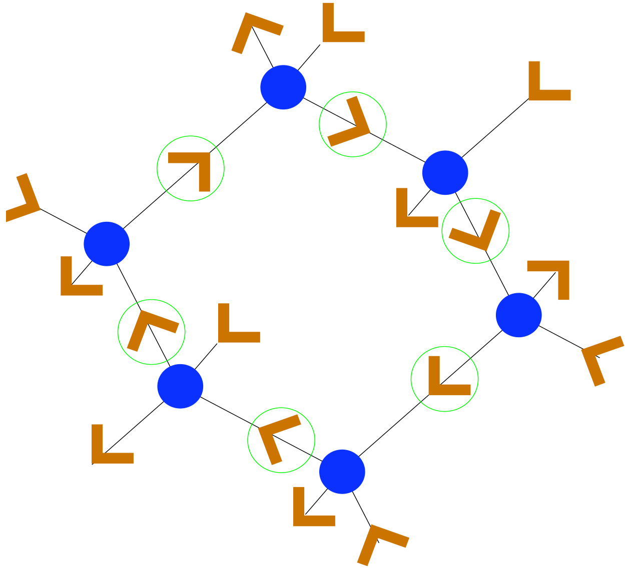


- Define 'flux' vector field on *links* of the ice lattice: B_i
- Local constraint (ice rules) becomes conservation law (as in Kirchoff's laws)

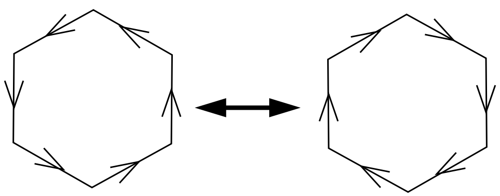
  $\Rightarrow$ gauge theory

$$\nabla \cdot \mathrm{B} = 0 \Longrightarrow \mathrm{B} = \nabla \times \mathbf{A}$$

- Spin ice configurations differ by flipping spins in a loop
- Amounts to reversing closed loop of flux B
- Smallest loop: hexagon (six links)

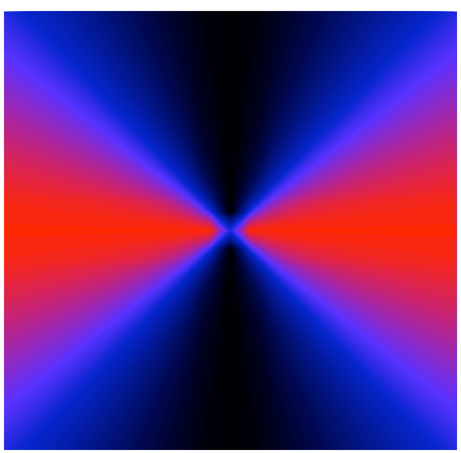


Long-wavelength analysis: coarse-graining



- Coarse-grain $\mathbf{B} \to \tilde{\mathbf{B}}$ with $\nabla \cdot \tilde{\mathbf{B}} = 0$
- 'Flippable' loops have zero average flux:
  low average flux $\Leftrightarrow$ many microstates
- Ansatz: upon coarse-graining, obtain energy functional of entropic origin:

$$Z = \sum_{\mathbf{B}} \delta_{\nabla.\mathbf{B},0} \to \int \mathcal{D}\tilde{\mathbf{B}}\; \delta(\nabla \cdot \tilde{\mathbf{B}})\; \exp[-\frac{K}{2}\tilde{\mathbf{B}}^2]$$

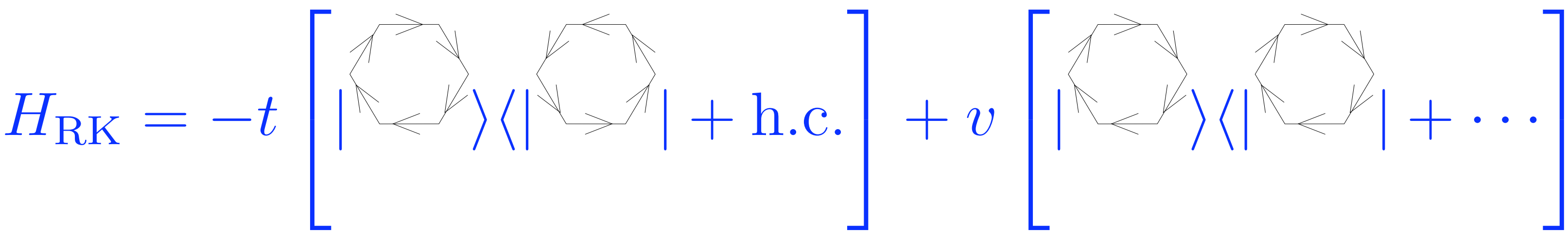


- Artificial magnetostatics!
- Resulting correlators are transverse and algebraic (but not critical!): e.g.

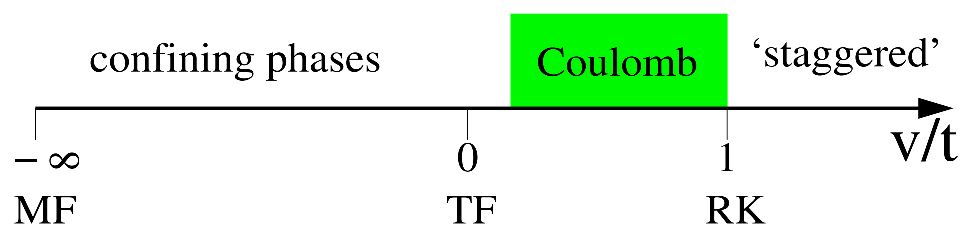
$$\langle \tilde{B}_z(q)\tilde{B}_z(-q)\rangle \propto q_\perp^2/q^2 \leftrightarrow (3\cos^2\theta - 1)/r^3.$$

Quantum frustration: $U(1)$ Coulomb phase

- Hilbert space: classical ground states of (spin) ice
- Add coherent quantum dynamics for hexagonal loop:



- Effective long-wavelength theory $\mathcal{S}_q = \int \vec{E}^2 - c^2 \vec{B}^2$ Maxwell
- This describes the Coulomb phase of a $U(1)$ gauge theory:
  - gapless photons, speed of light $c^2 \propto t - v$
  - deconfinement
  - microscopic model!



- Artificial electrodynamics with frustrated system as 'ether'

Collaborators

Theory:

- Claudio Castelnovo (Oxford)
- John Chalker (Oxford)
- Karol Gregor (Caltech)
- Sergei Isakov (ETHZ)
- Kumar Raman (UIUC)
- Shivaji Sondhi (Princeton)
- Adam Willans (Oxford)

Experiment:

- Steve Bramwell (UCL)
- Zenji Hiroi (Tokyo)
- Art Ramirez (Alcatel-Lucent)
- Peter Schiffer (Penn State)

+ many more

Fractionalisation in simple lattice models

Triangular lattice correlated electrons

- spin-charge separation
- topological order $\Rightarrow$ quantum computing

Fractionalisation in spin model in $d = 3$

- n.n. or dipolar Ising model with non-collinear axes
- frustration gets rid of simple ordered ground state
  - huge low-temperature entropy
  - monopoles (also classically) and artificial photons (qm)
  - algebraic correlations without criticality
  - several ways of obtaining dimensional reduction