

Problem set: Cooling fermions in a trap

(Dated: Oxford summer school 2008)

In this problem we will consider a cooling method of cold fermions by means of a Feshbach resonance. The anticipated cooling sequence consists of the following steps:

1. Prepare a cold gas of N_F fermions in a harmonic trap in an equal spin mixture of spin-up and spin-down states on the attractive side ($a < 0$) of a Feshbach resonance at temperature $T < T_F$. The Fermi temperature for each of the spin components is $k_B T_F = \hbar\omega(3N_F)^{1/3}$ and the entropy is $S_F = N_F \pi^2 k_B T/T_F$.
2. We ramp the magnetic field across the Feshbach resonance and adiabatically convert a fraction η of the atoms into (bosonic) molecules. The critical temperature for N_m bosons is $k_B T_c = \hbar\omega(N_m/\zeta(3))^{1/3}$ and the entropy for $T/T_c < 1$ is $S_B = N_m k_B \frac{2\pi^4}{45\zeta(3)} (T/T_c)^3$.
3. After having created the molecules we remove the remaining unpaired atoms with a pulse of laser light from the trap (non-adiabatic).
4. Finally, we ramp the magnetic field back across the Feshbach resonance and adiabatically convert the molecules back into atoms.

Questions:

1. Explain how the cooling mechanism works. Sketch the contribution to the total entropy of both atoms and molecules.
2. Calculate the final temperature $(T/T_F)_{final}$ under the assumption that the ramps across the Feshbach are adiabatic (entropy is conserved). Ignore all effects of interactions on T_c and entropy (the use of Mathematica is not necessary but could be helpful).
3. Explain why the two extreme cases ($\eta = 0$ and $\eta = 1$) represent the least efficient cooling path.
4. As discussed in the lecture, the probability of molecule formation is temperature dependent. Taking this into account, discuss the efficiency of the cooling process.

For further reading: L. D. Carr, R. Chiamonte, M. J. Holland, Phys. Rev. A 70, 043609 (2004).