Fermionic Quantum Gases in Optical Lattices

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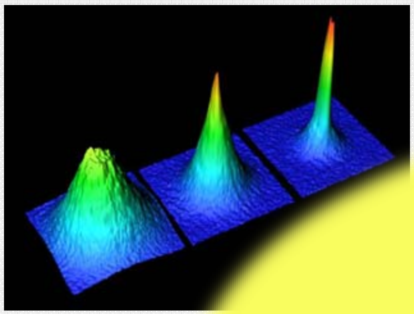
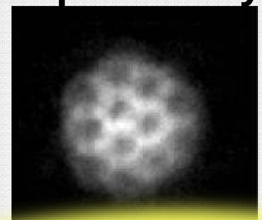
Condensed matter physics with cold gases

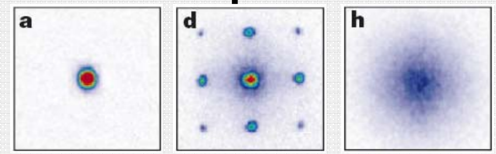
Bose-Einstein condensation



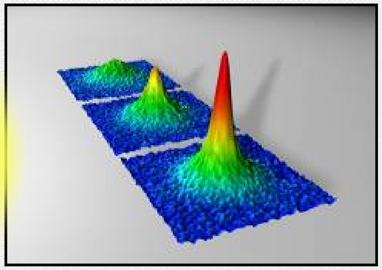
Superfluidity



Bosons in optical lattices

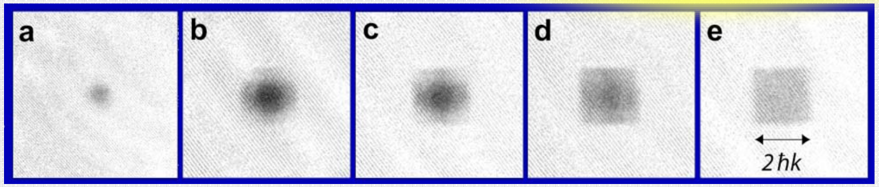


Strong interactions
(e.g. BEC-BCS crossover)

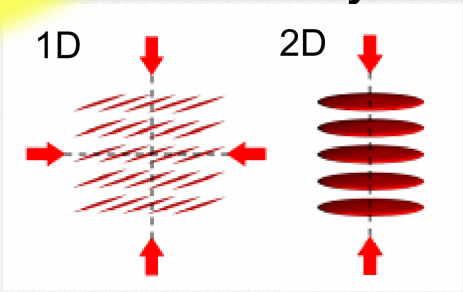


- Exceptional experimental control & tunability
- Strong correlations
- New physics envisioned

Fermionic atoms in



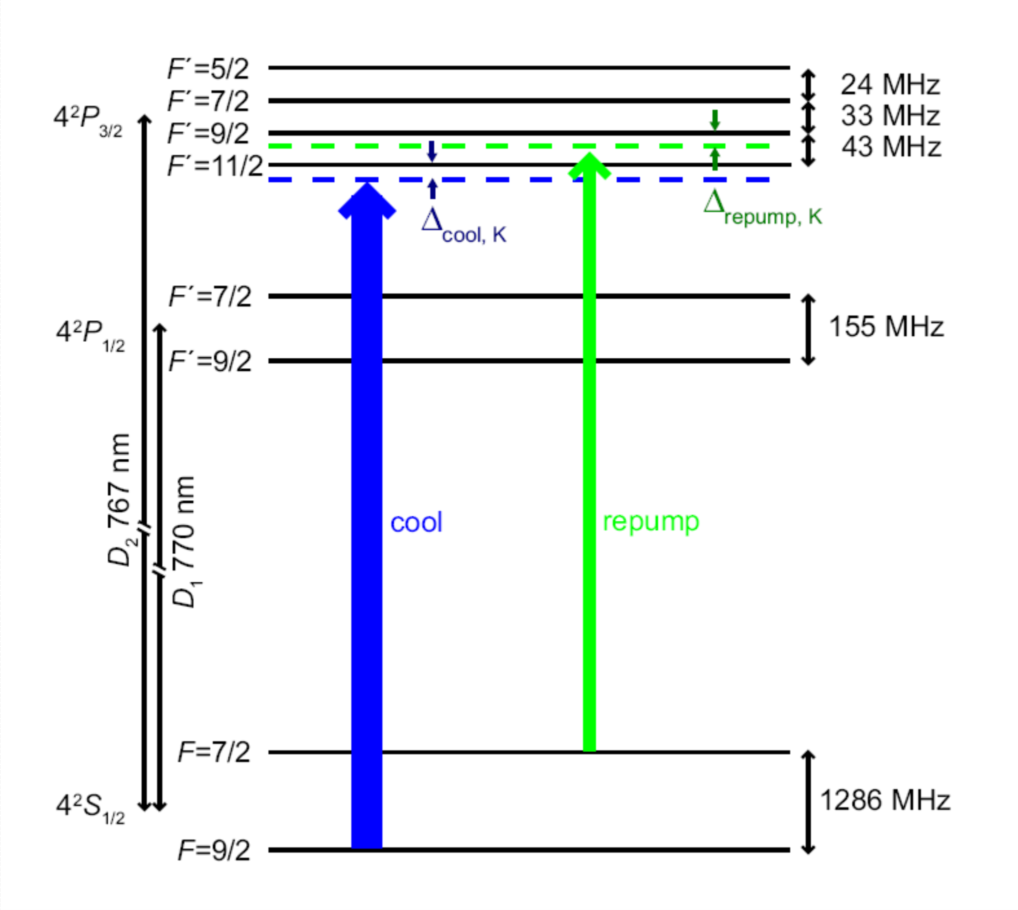
dimensional systems



Quantum degenerate Fermi gases

Laser cooling

^{40}K; I=4



Two stable fermionic isotopes:

- ^{6}Li (671 nm)
- ^{40}K (767 nm)

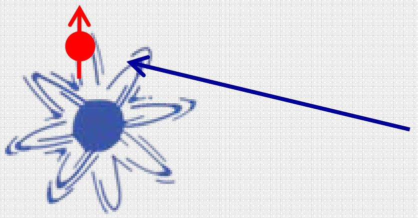
- Laser cooling is more difficult (small excited state hyperfine splitting, only few times linewidth)

- sub-Doppler cooling is impossible for Li

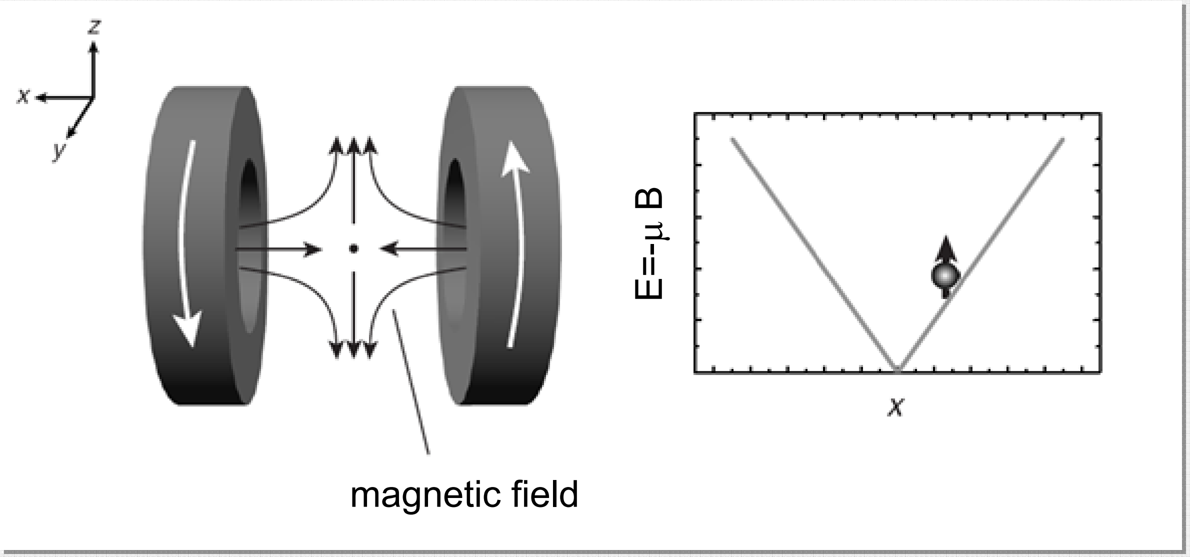
- relatively big atoms number can be achieved in a MOT (10^8-10^9)

The laser setup

Magnetic trapping



atom has a magnetic moment μ
(due to the valence electron)



Fermions at low temperatures

total wave function is antisymmetric for fermions:

$$|\psi\rangle = |\psi_{spatial}\rangle \otimes |\psi_{spin}\rangle$$

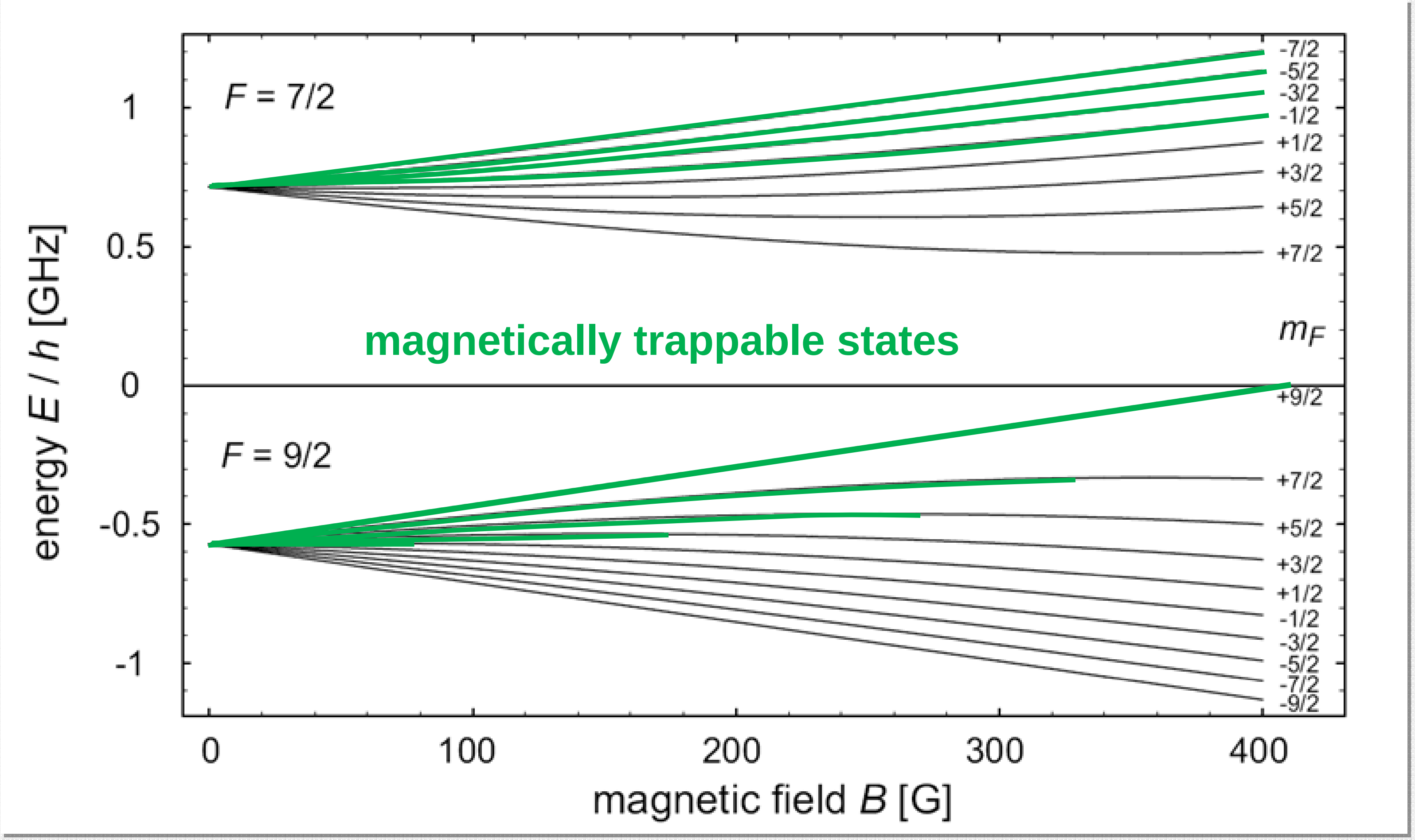
no s-wave collisions possible
(pair correlation length: λ_{dB})

$\psi_{spatial}$	ψ_{spin}	
anti-symmetric	$|\uparrow\uparrow\rangle$	spin-polarized
anti-symmetric	$|\downarrow\downarrow\rangle$	spin-polarized
anti-symmetric	$|\uparrow\downarrow + \downarrow\uparrow\rangle$	spin mixture
symmetric	$|\uparrow\downarrow - \downarrow\uparrow\rangle$	spin mixture

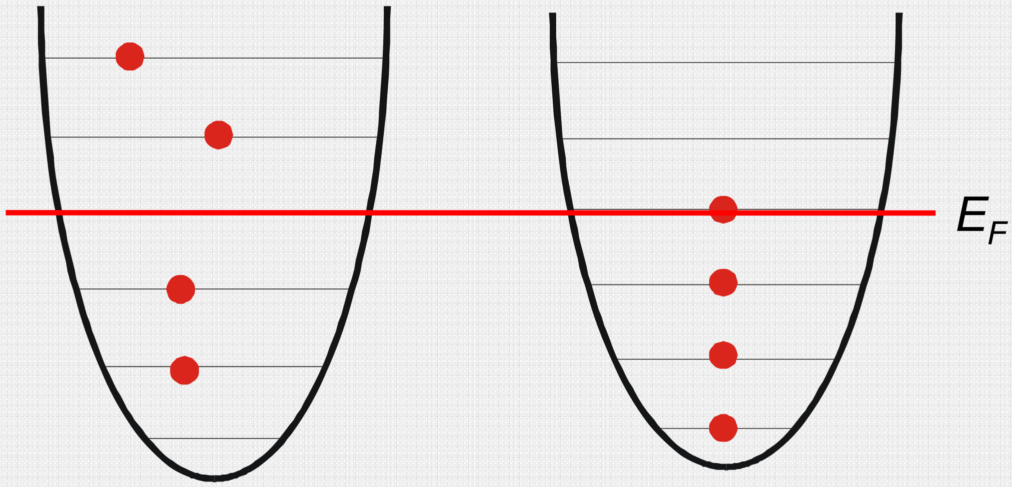
Cooling fermions is more difficult

- **use mixture of different spin states**
- **use mixture with a different species, e.g. bosons**

Magnetic levels of potassium



Fermions in a harmonic trap

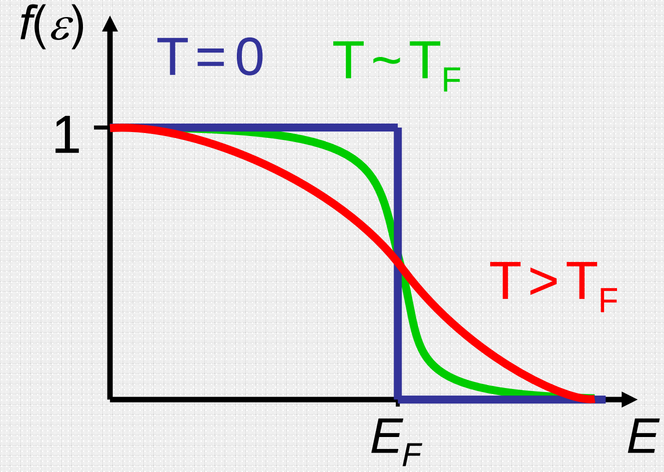


Fermi-Dirac distribution

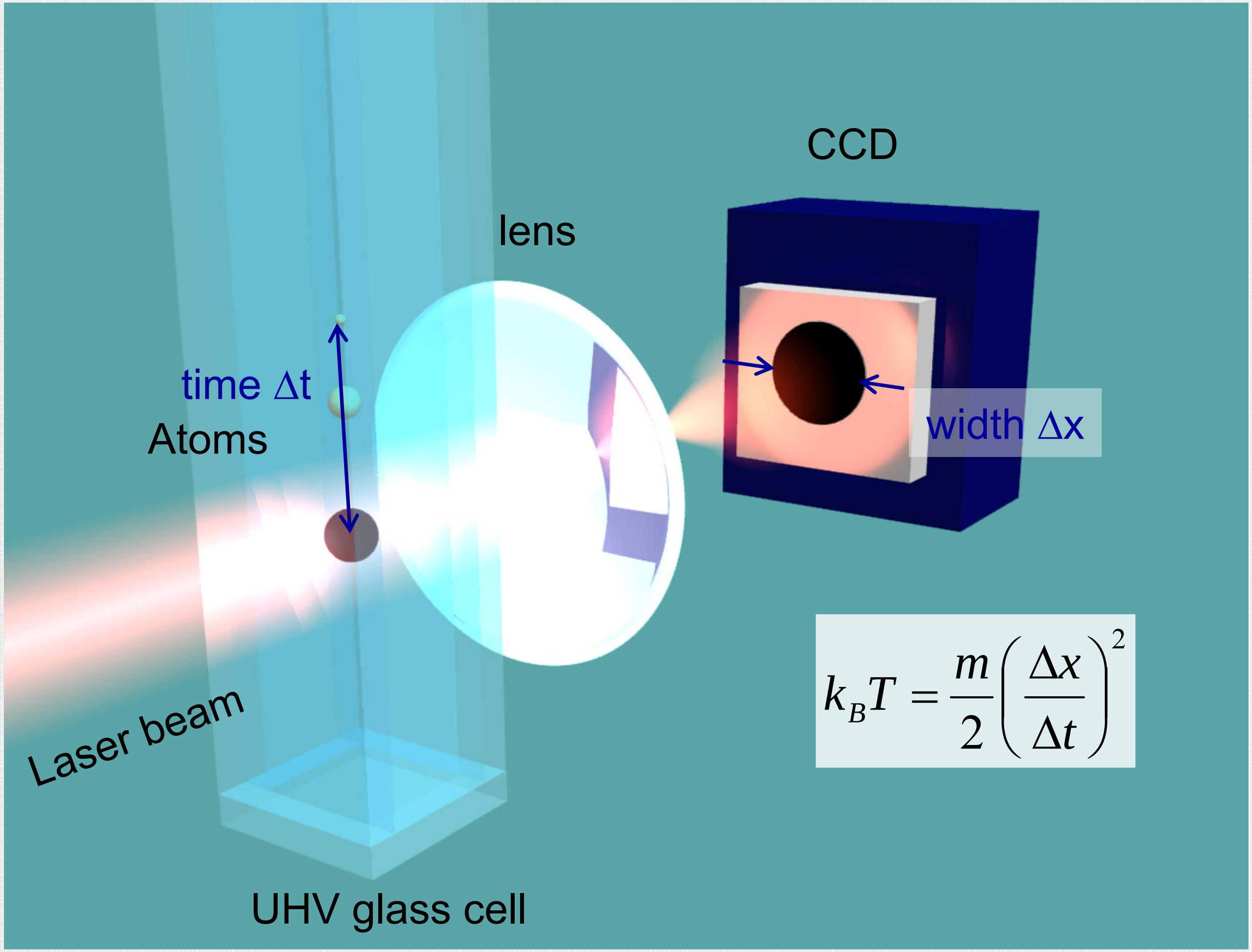
$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$$

Fermi temperature

$$kT_F = \hbar\omega(6N)^{1/3}$$



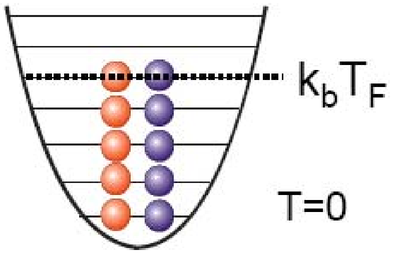
How to measure density and temperature

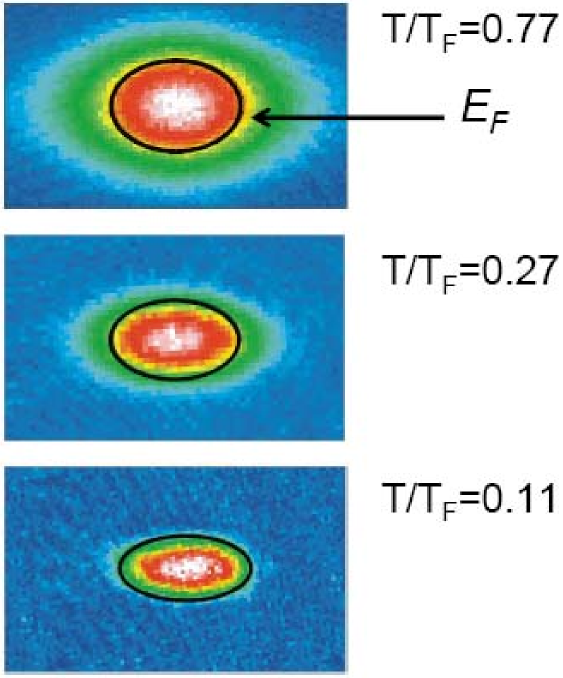


$$k_B T = \frac{m}{2}\left(\frac{\Delta x}{\Delta t}\right)^2$$

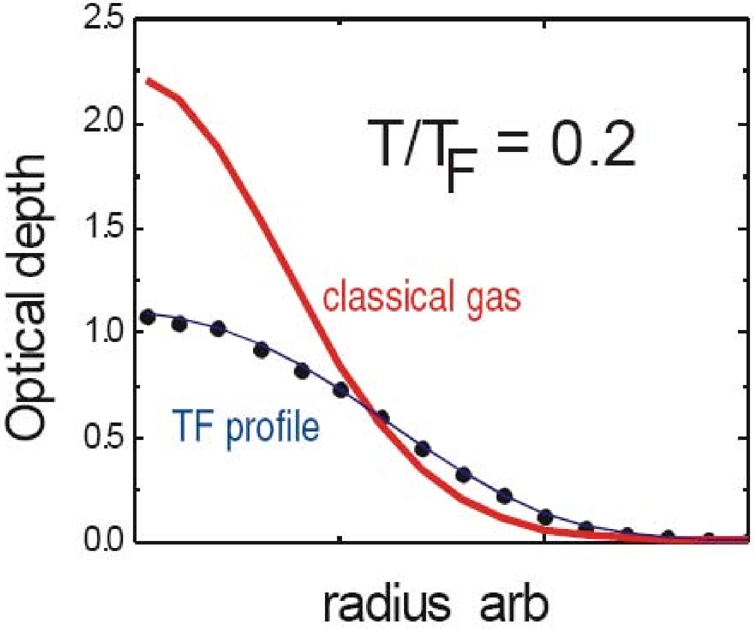
Fermi degeneracy

Fermi degeneracy

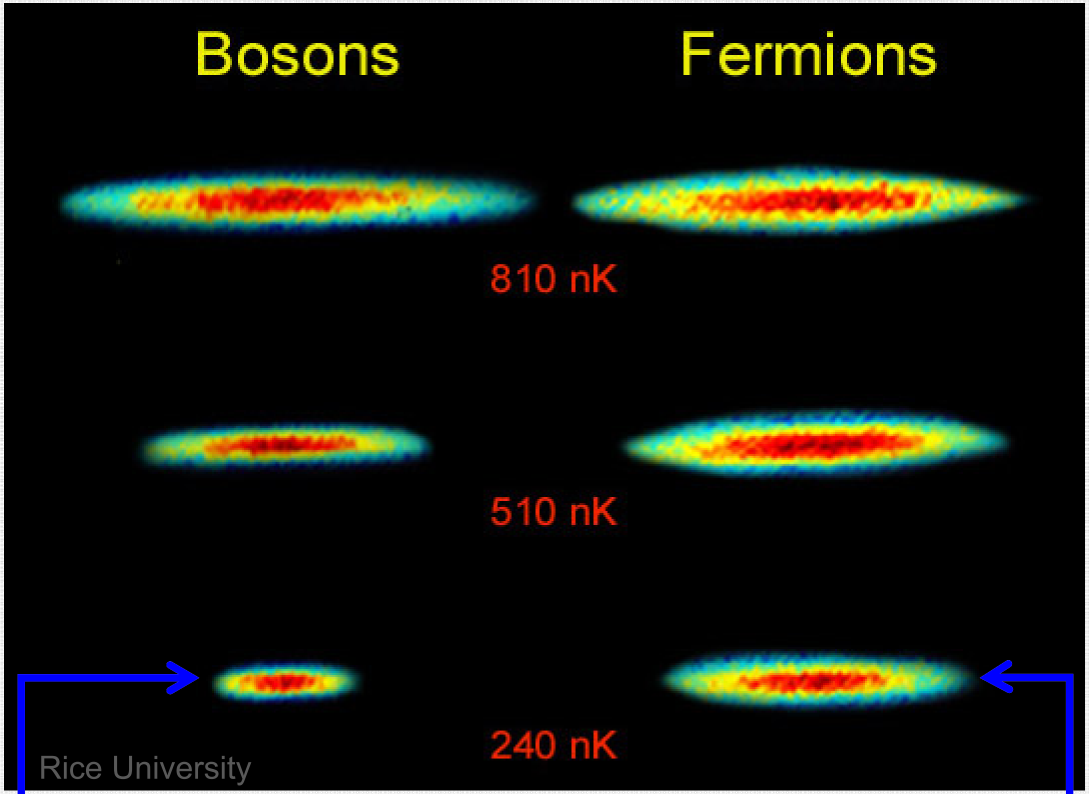




(Regal et al., JILA)



Bosons vs. Fermions



Bosons shrink to ground state

Pauli's exclusion principle prevents fermions from shrinking

Fermions in optical lattices

Generating periodic potentials

Induced electric dipole potential: $V = -\frac{1}{2}\alpha\left|E\right|^2$

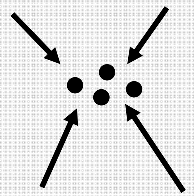
ac polarizability of the atom

electric field of the laser

Two options:

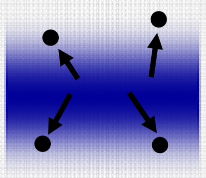
$\omega_L < \omega_A$

„red detuned“

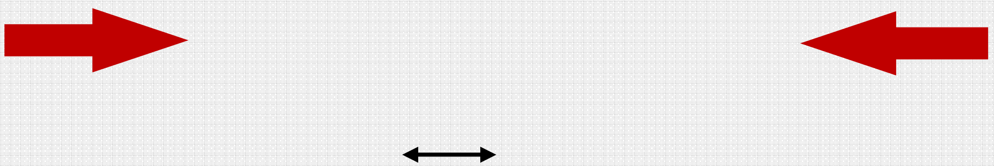


$\omega_L > \omega_A$

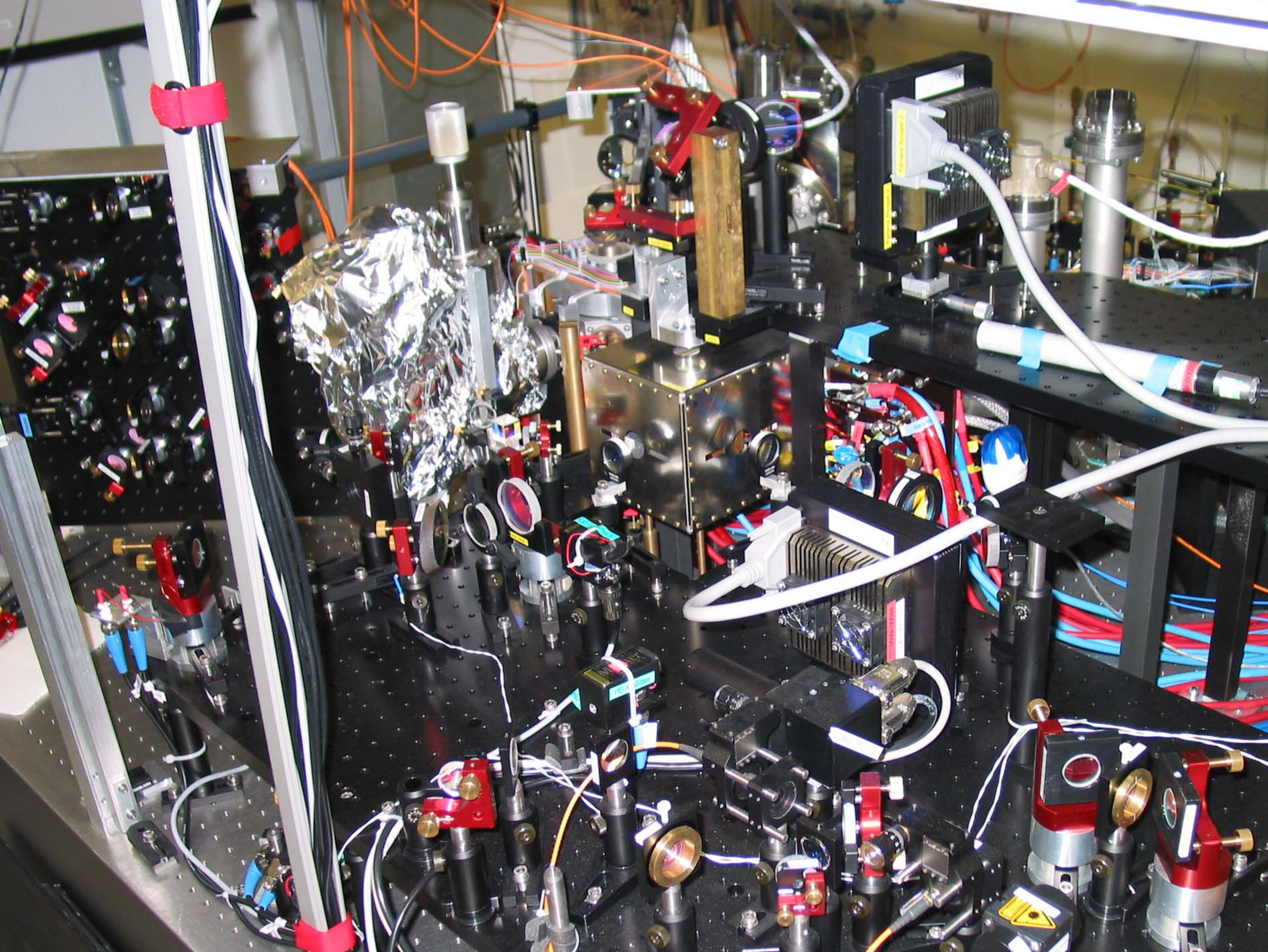
„blue detuned“



Optical lattice



$\lambda / 2 \approx 400\ nm$



Optical lattice potentials

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U\sin^2\left(\frac{2\pi}{\lambda}x\right)$$

scales in units of the photon recoil energy: $E_{rec} = \hbar^2 k^2 / 2m$

intensity

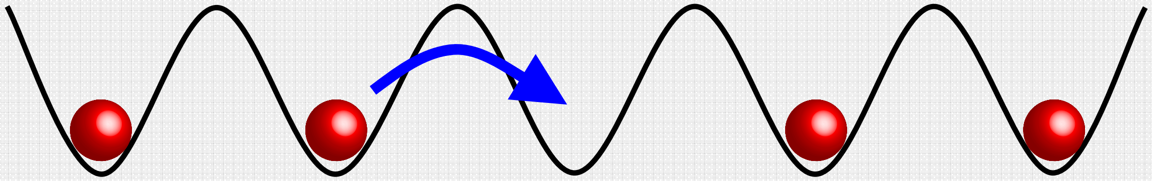
dipole matrix element

Potential depth: $U \propto \dfrac{I\left|\left\langle e\middle|\hat{D}\middle|g\right\rangle\right|^2}{\Delta}$

detuning from resonance

	^{6}Li	^{40}K
532nm, 1 W, 150 μm	U = -10 μK = -2 E_{rec}	U = -10 μK = -12 E_{rec}
850 nm, 200 mW, 150 μm	U = 3 μK = 1.5 E_{rec}	U = 12 μK = 36 E_{rec}
1064 nm, 1W, 150 μm	U = 10μK = 7 E_{rec}	U = 23 μK = 110 E_{rec}

Fermions in a lattice



tunneling

$$-J\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i-1,\sigma}$$

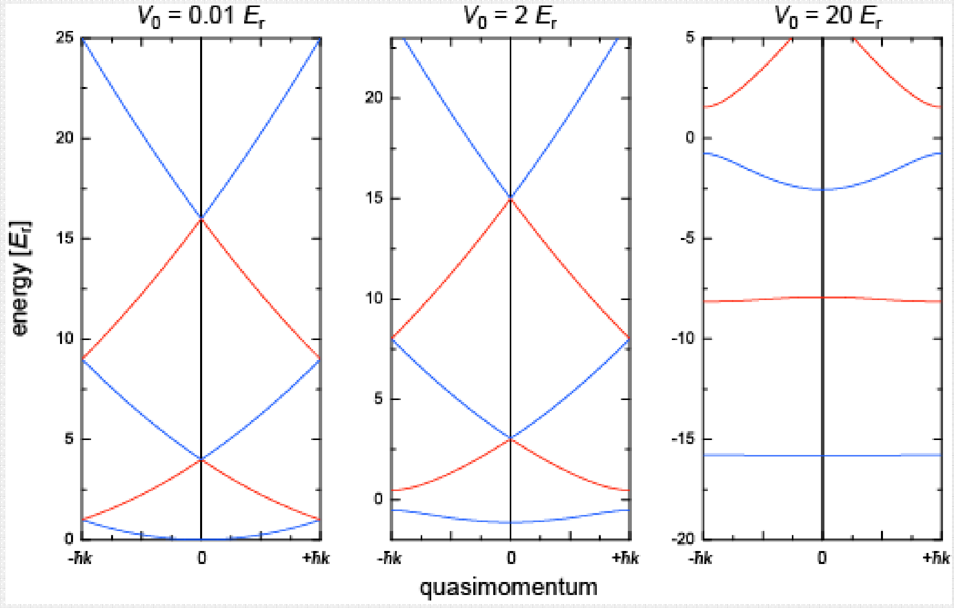
$$H = -J \sum_{<i,j>,\sigma} c^{+}_{i,\sigma} c_{j,\sigma}$$

$$\frac{J}{E_{rec}} = \frac{4}{\sqrt{\pi}} s^{3/4} \exp\left(-2\sqrt{s}\right)$$

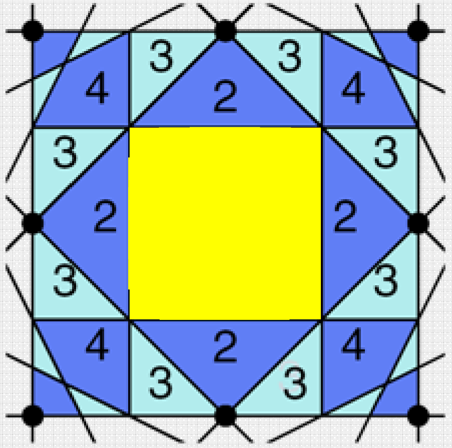
$s = U_{lattice}/E_{rec}$ lattice depth

Ideal Fermi gas in a lattice

Band structure



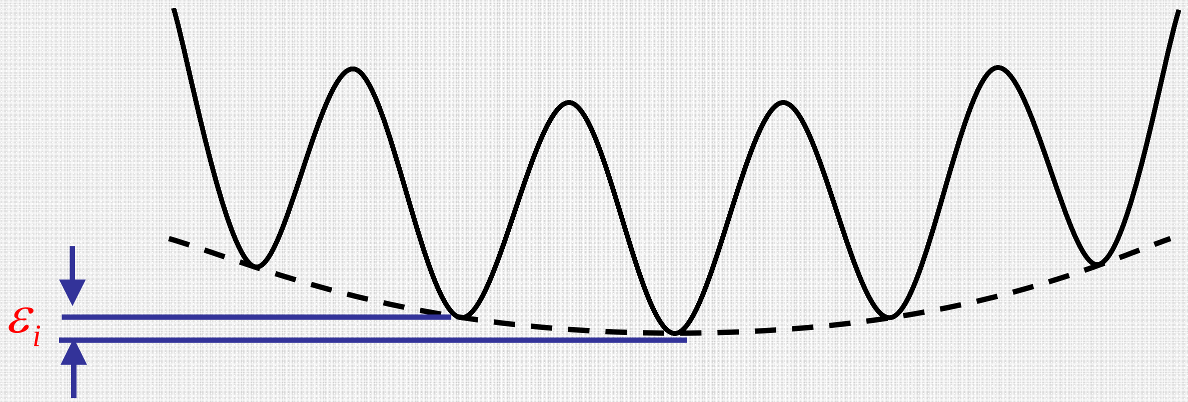
Brillouin zones
of a square lattice

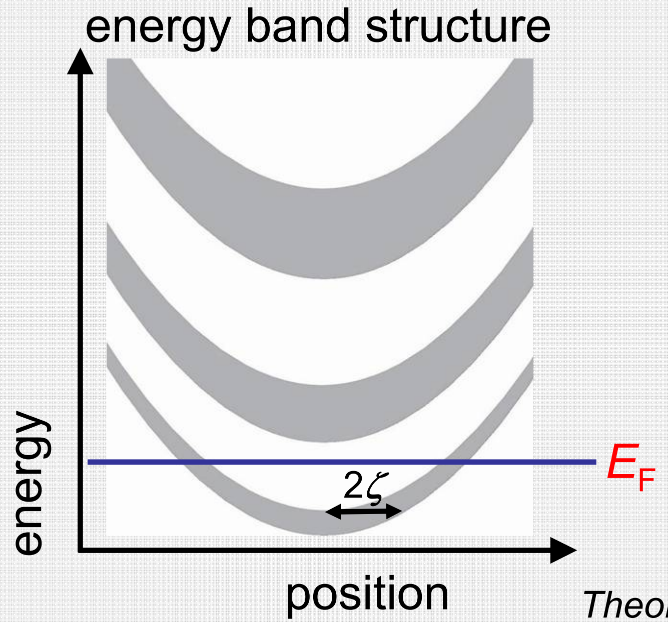


quasi momentum

2 ħk

The inhomogeneous lattice





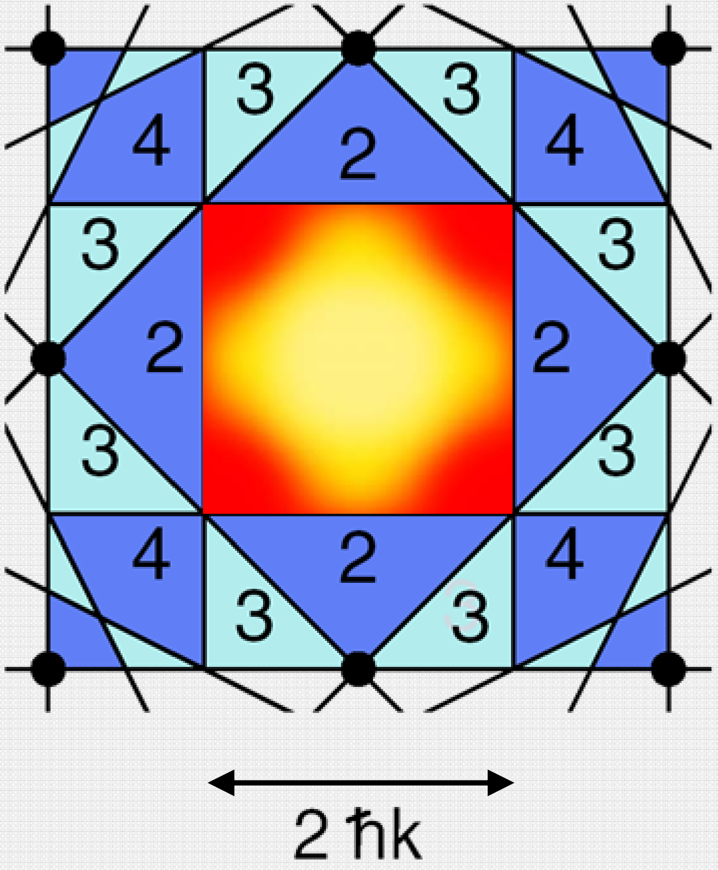
characteristic length scale $\zeta = \sqrt{\frac{2J}{m\omega^2}}$

characteristic density $\rho_c = \frac{N}{(\zeta/d)^D}$

N: particle number,
D: dimension
d: lattice constant

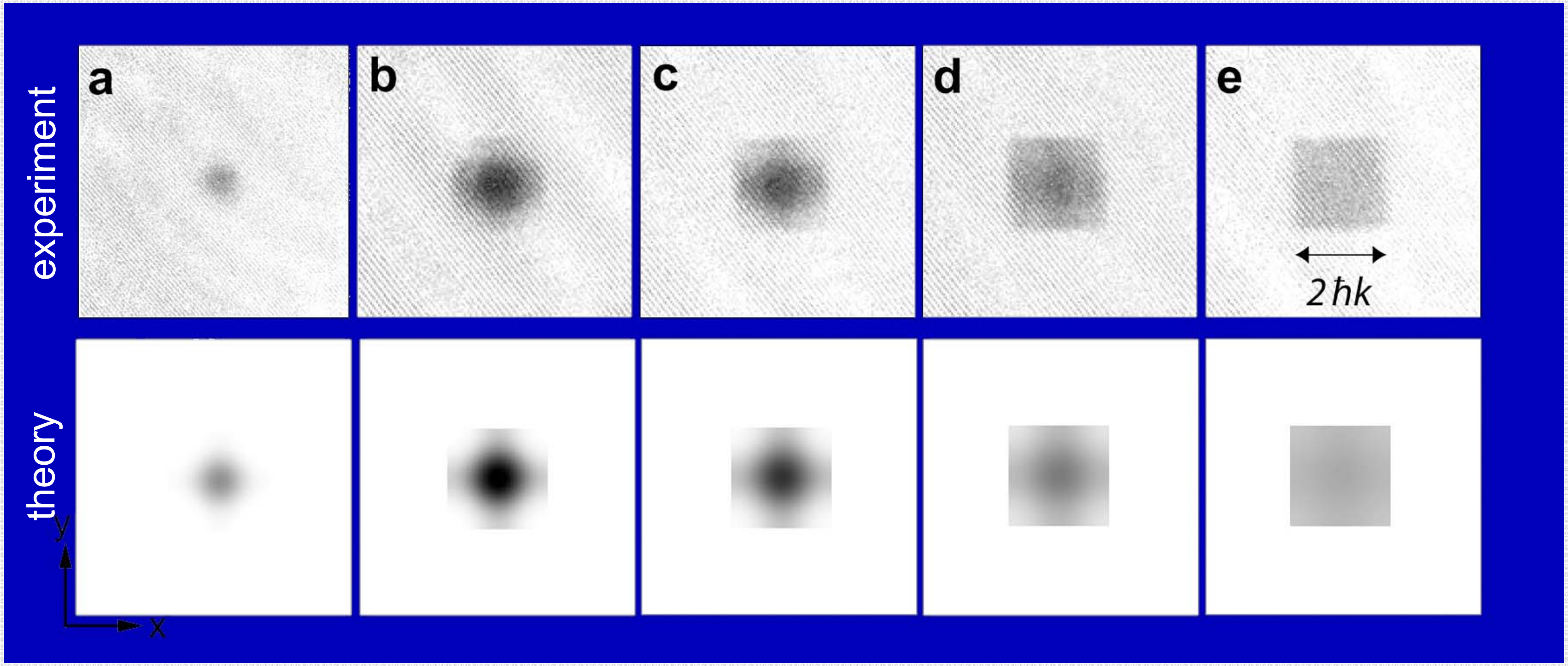
Theory: M. Rigol and A. Muramatsu Phys. Rev. A 69, 053612 (2004)

Fermi Surface in the inhomogeneous lattice



quasi momentum

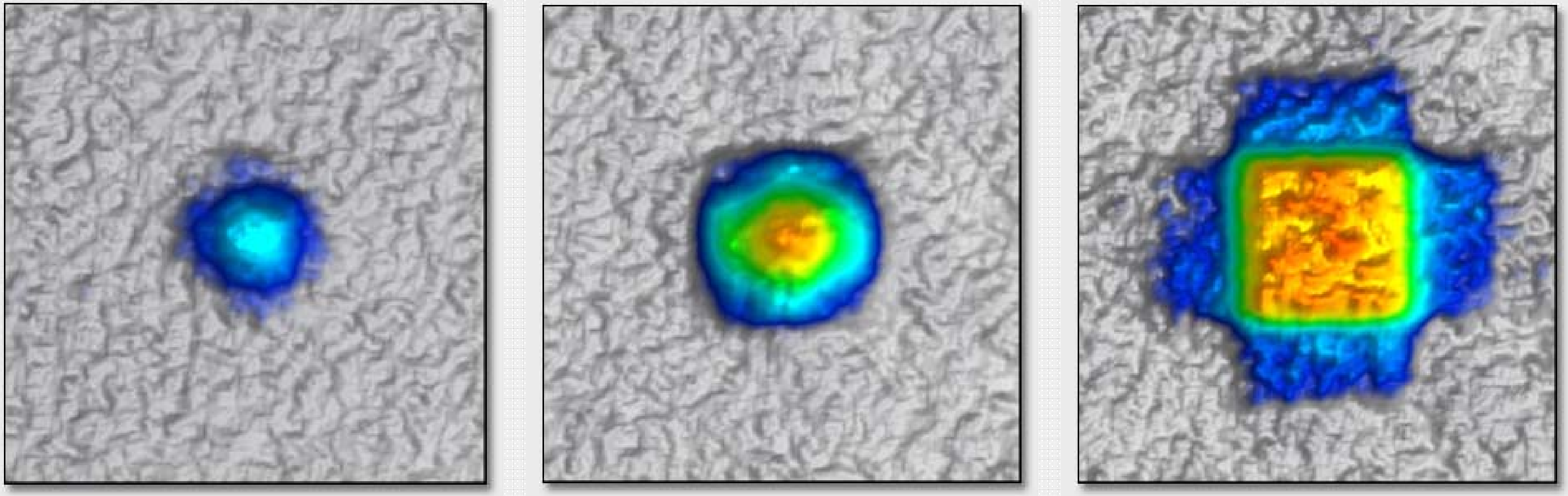
Observed Fermi surfaces



“conductive state“ → filling → “band insulator“

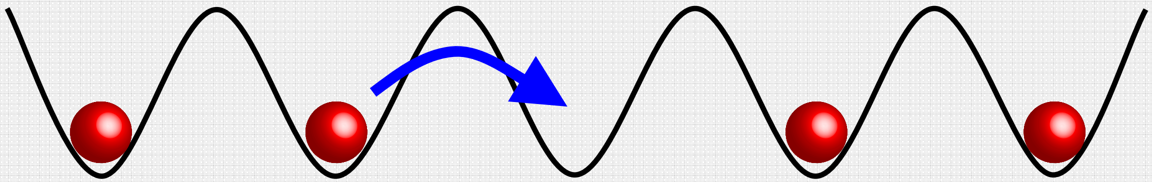
M. Köhl et al., Phys. Rev. Lett. 94, 080403 (2005).

Experiment: Fermi surfaces



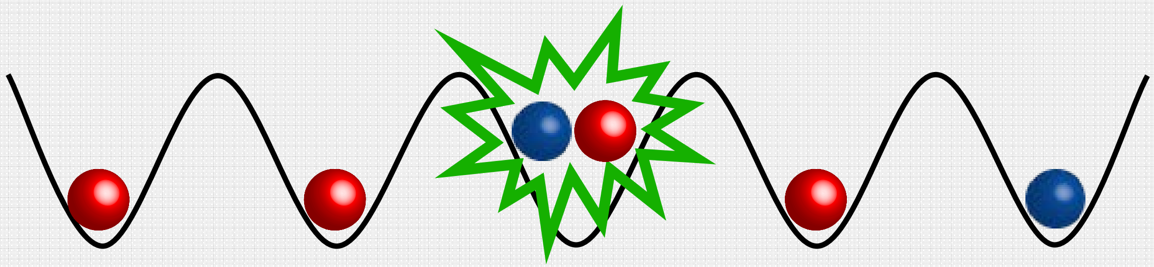
Interacting Fermi gases

Fermions in a lattice



tunneling

$$-J\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i-1,\sigma}$$



interaction

$$U\hat{n}_{i\uparrow}\hat{n}_{i\downarrow}$$

$$H = -J\sum_{<i,j>,\sigma}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U\sum_{i}\hat{n}_{i\uparrow}\hat{n}_{i\downarrow} - \sum_{i,\sigma}(\mu-\varepsilon_{i,\sigma})\hat{n}_{i,\sigma}$$

$$\frac{U}{E_{rec}} = \sqrt{\frac{8}{\pi}}kas^{3/4}$$

„Hubbard model“

Basic properties of the Fermi-Hubbard model

Two spin ½ fermions on two sites

$$\left|\uparrow\downarrow,0\right\rangle, \left|\uparrow,\downarrow\right\rangle, \left|\downarrow,\uparrow\right\rangle, \left|0,\uparrow\downarrow\right\rangle$$

left site   right site

attractive U:

$$\left|\psi\right\rangle \approx \left|\uparrow\downarrow,0\right\rangle + \left|0,\uparrow\downarrow\right\rangle + \frac{J}{2U}\left(\left|\uparrow,\downarrow\right\rangle - \left|\downarrow,\uparrow\right\rangle\right)$$

$$\left\langle\psi\middle|n_\uparrow n_\downarrow\middle|\psi\right\rangle \approx 1$$

- particles tend to pair
- superfluid

repulsive U:

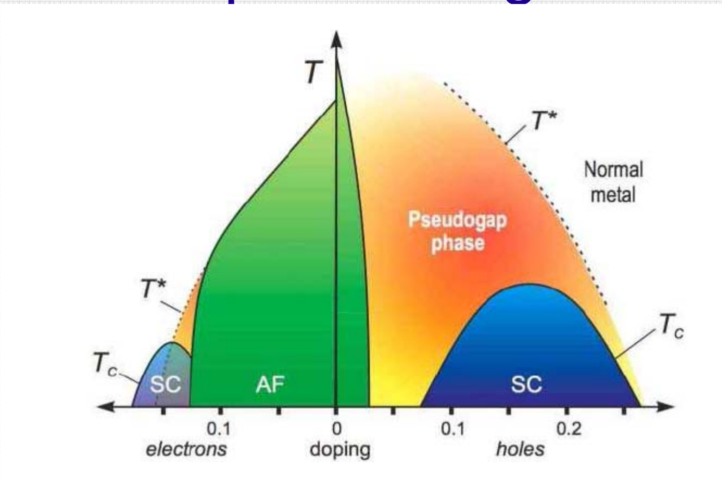
$$\left|\psi\right\rangle \approx \left|\uparrow,\downarrow\right\rangle - \left|\downarrow,\uparrow\right\rangle + \frac{J}{2U}\left(\left|\uparrow\downarrow,0\right\rangle + \left|0,\uparrow\downarrow\right\rangle\right)$$

$$\left\langle\psi\middle|n_\downarrow n_\uparrow\middle|\psi\right\rangle \propto \frac{J}{U}$$

- particles order with alternating spin
- insulator (anti-ferromagnet)

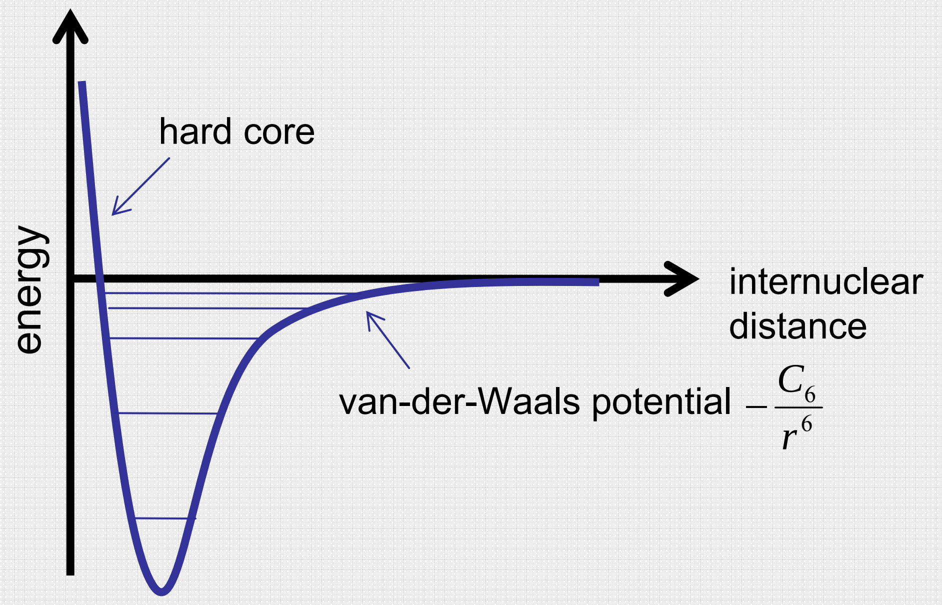
More particles ...

- The ground state is known only for special cases, e.g.:
  attractive U: superfluid/superconductor
  repulsive U, half-filling ($N_\uparrow = N_\downarrow = M/2$): anti ferromagnet
  one dimension, infinite dimensions
- Sketch of a possible phase diagram for U>0:



- The general problem cannot be solved theoretically: the Hilbert space is too big!

Interactions between neutral atoms



Basic properties of ultracold collisions

- What is the range of a power-law potential $-\frac{C_n}{r^n}$ ($n>2$)?

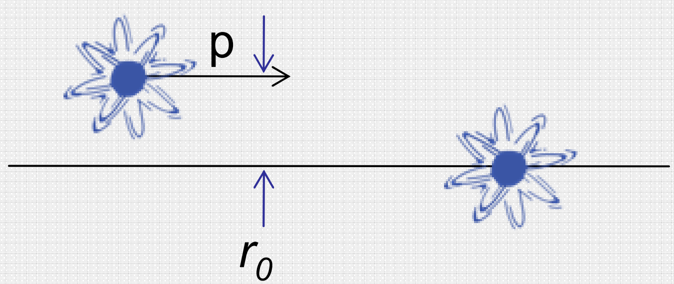
a particle prepared at a distance r has a kinetic energy (Heisenberg limit due to the hard core) $E_{kin} \geq \frac{\hbar^2}{m r^2}$

potential energy of the particle $E_{pot} = \frac{C_n}{r^n}$

$$E_{kin} = E_{pot} \quad @ \quad r_0 = \left(\frac{mC_n}{\hbar^2} \right)^{\frac{1}{n-2}}$$

typical values for alkali atoms: $r_{eff} \approx 10^2\, a_{Bohr}$

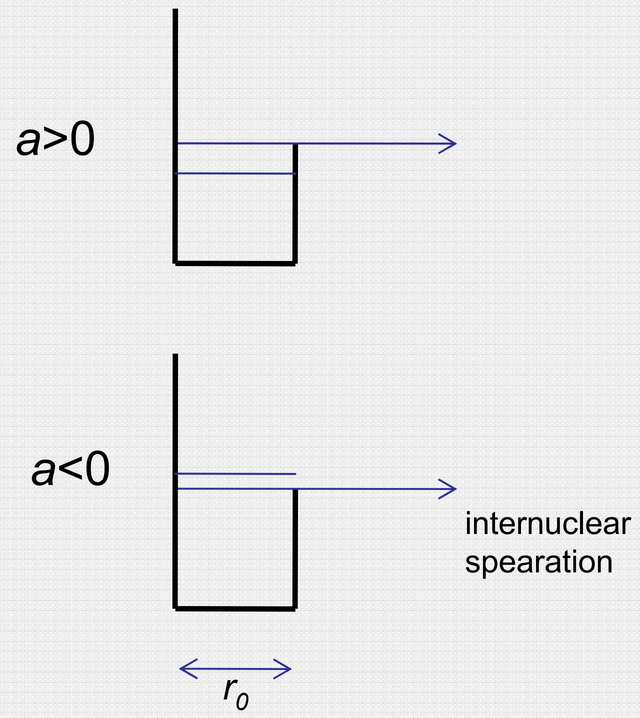
- s-wave collisions



$$p \approx \sqrt{mkT}$$

$$p \cdot r_0 < \hbar \quad \rightarrow \quad T < 100\,\mu K$$

Basics of low-energy scattering theory

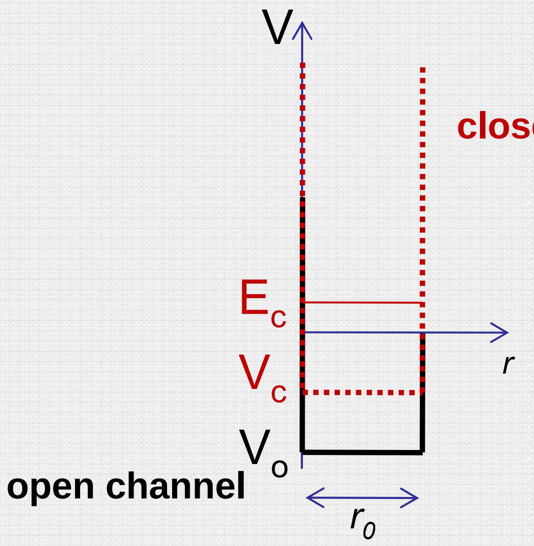


Interaction strength in the Born approximation

$$U_{\text{int}} = \frac{4\pi\hbar^2 a}{m}$$

depends on ***a*** but not on the potential.

A toy model for a Feshbach resonance



$$H = -\frac{\hbar^2}{m}\nabla^2 + V$$

$$V = \begin{cases} -\begin{pmatrix} V_o & \Omega \\ \Omega & V_c \end{pmatrix} & r < r_0 \\ \begin{pmatrix} 0 & 0 \\ 0 & \infty \end{pmatrix} & r > r_0 \end{cases}$$

hyperfine interaction: $\Omega < \{V_o, V_c, |V_o - V_c|\}$

Ansatz:

$$|\psi\rangle = \psi_o(r)|o\rangle + \psi_c(r)|c\rangle$$

open channel — closed channel

A toy model for a Feshbach resonance

Solution for E=0

$$\text{for } r > r_0 : |\psi\rangle \;\propto\; \frac{r-a}{r}|o\rangle,$$

$$\text{for } r < r_0 : |\psi\rangle \;\propto\; \frac{\sin q_+ r}{r}|+\rangle + \frac{A \sin q_- r}{r}|-\rangle$$

$$|+\rangle = \cos\theta|o\rangle + \sin\theta|c\rangle$$

$$|-\rangle = -\sin\theta|o\rangle + \cos\theta|c\rangle$$

$$\tan 2\theta = \frac{2\Omega}{V_o - V_c} << 1$$

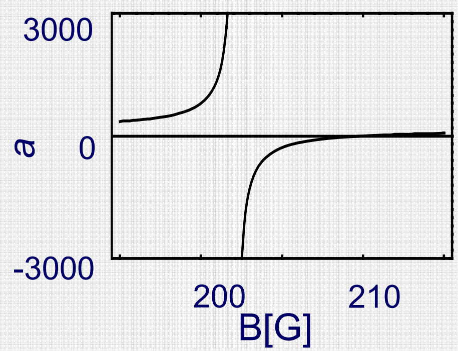
... the usual formalism: boundary conditions, continuity of the wavefunction, ...

magnetic field tuning

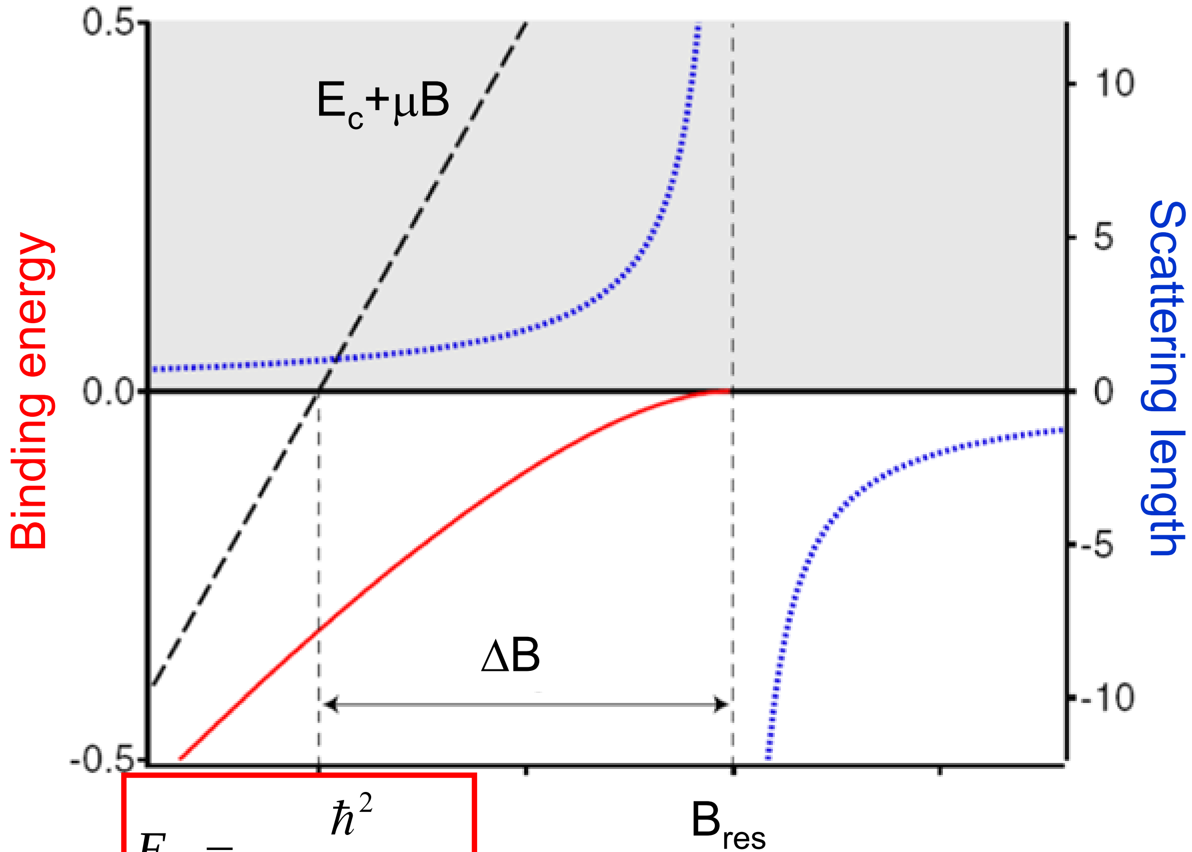
$$\frac{a - r_0}{a_{bg} - r_0} = \frac{E_c + \mu B}{E_c + \mu B + (a_{bg} - r_0)\frac{2V_c\theta^2}{r_0}} = 1 + \frac{\Delta B}{B - B_{res}}$$

Shift of the resonance position from where E_c crosses into the continuum of the open channel

Resonance of the scattering length !



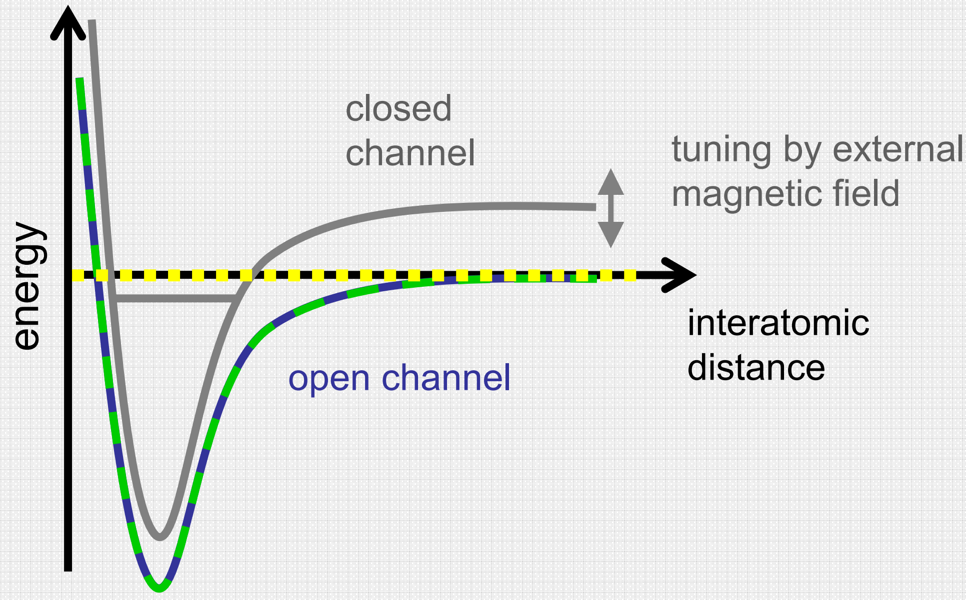
Feshbach molecules

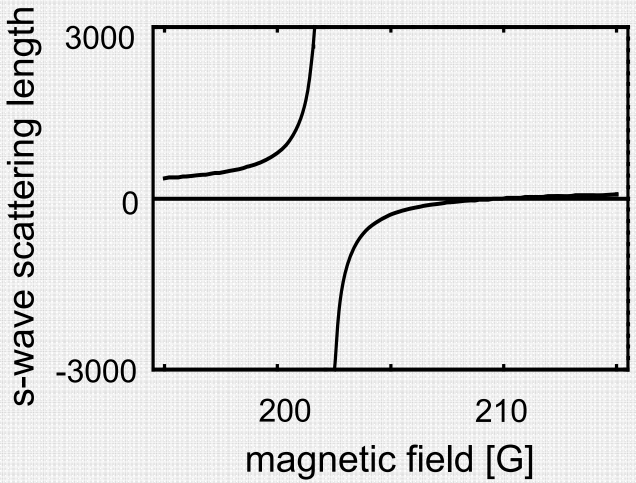


$$E_m = \frac{\hbar^2}{m\,(a - r_0)^2}$$

M.Zwierlein, PhD thesis

The more realistic picture





For ^{40}K

s-wave:

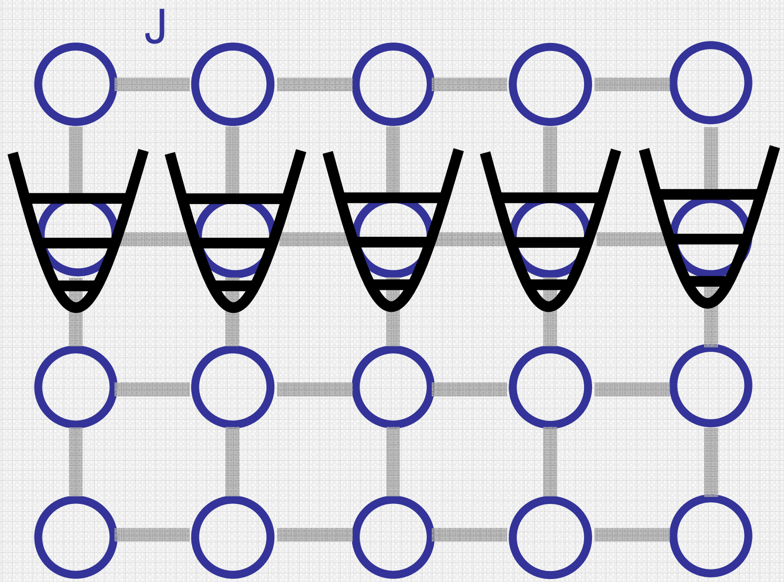
|9/2,-9/2> + |9/2,-7/2> @ 202 G

|9/2,-9/2> + |9/2,-5/2> @ 224 G

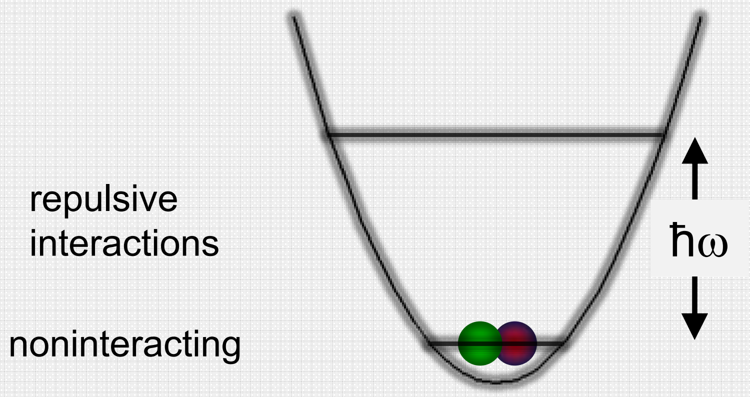
p-wave:

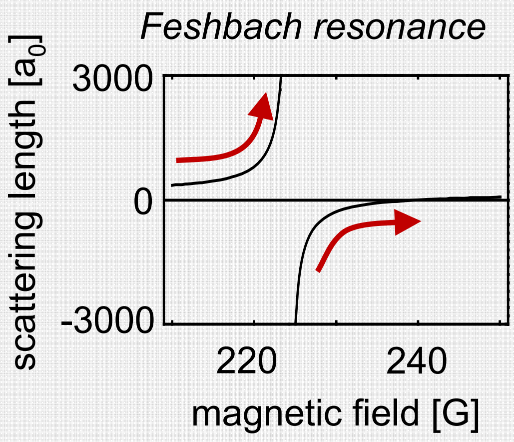
|9/2,-7/2> + |9/2,-7/2> @ 198 G

Strong interactions in the lattice



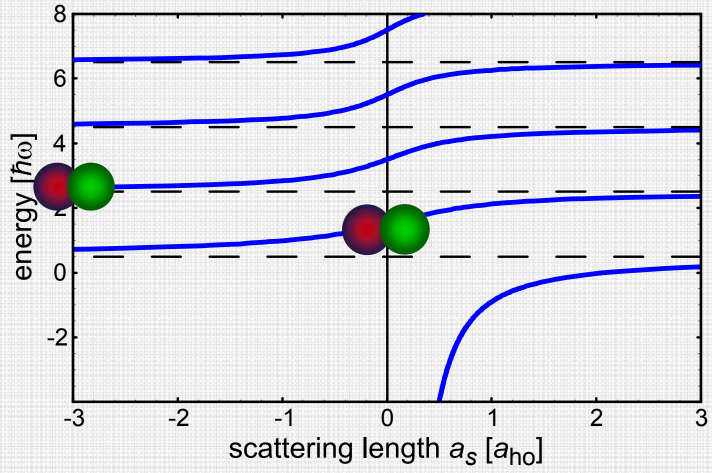
Repulsive interactions in a harmonic well





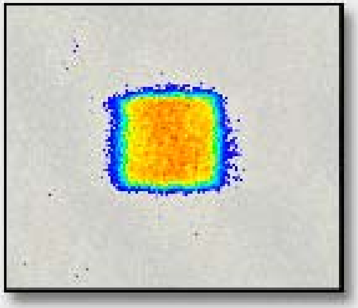
Theory according to
T. Busch et al., Found. Phys. 28, 549 (1998).

$$\frac{a_{ho}}{a} = \sqrt{2}\,\frac{\Gamma(-E_B/2\hbar\omega)}{\Gamma(-E_B/2\hbar\omega - 1/2)}$$

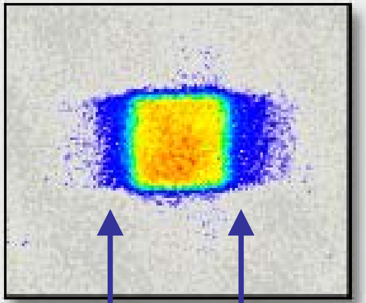


Coupling of the Bloch bands

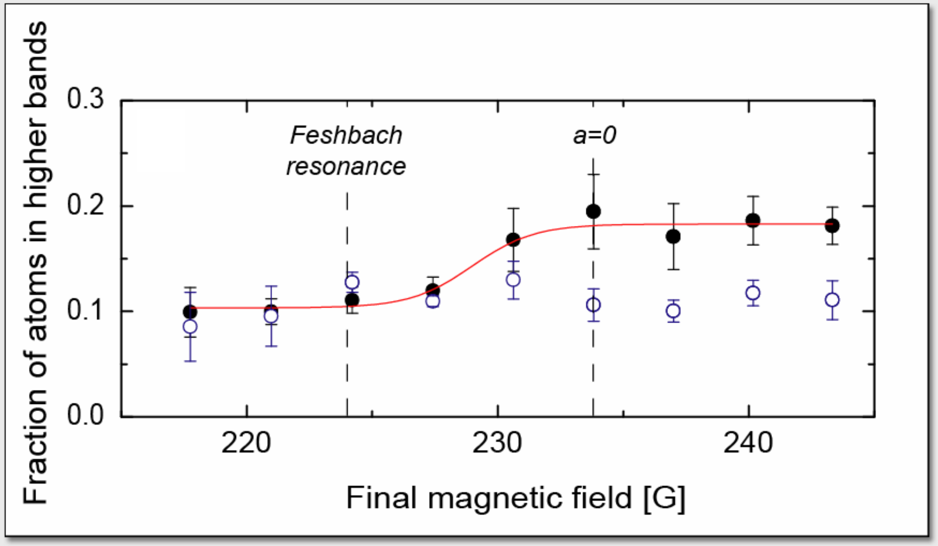
noninteracting



sweep over
Feshbach resonance



observe atoms in
higher bands



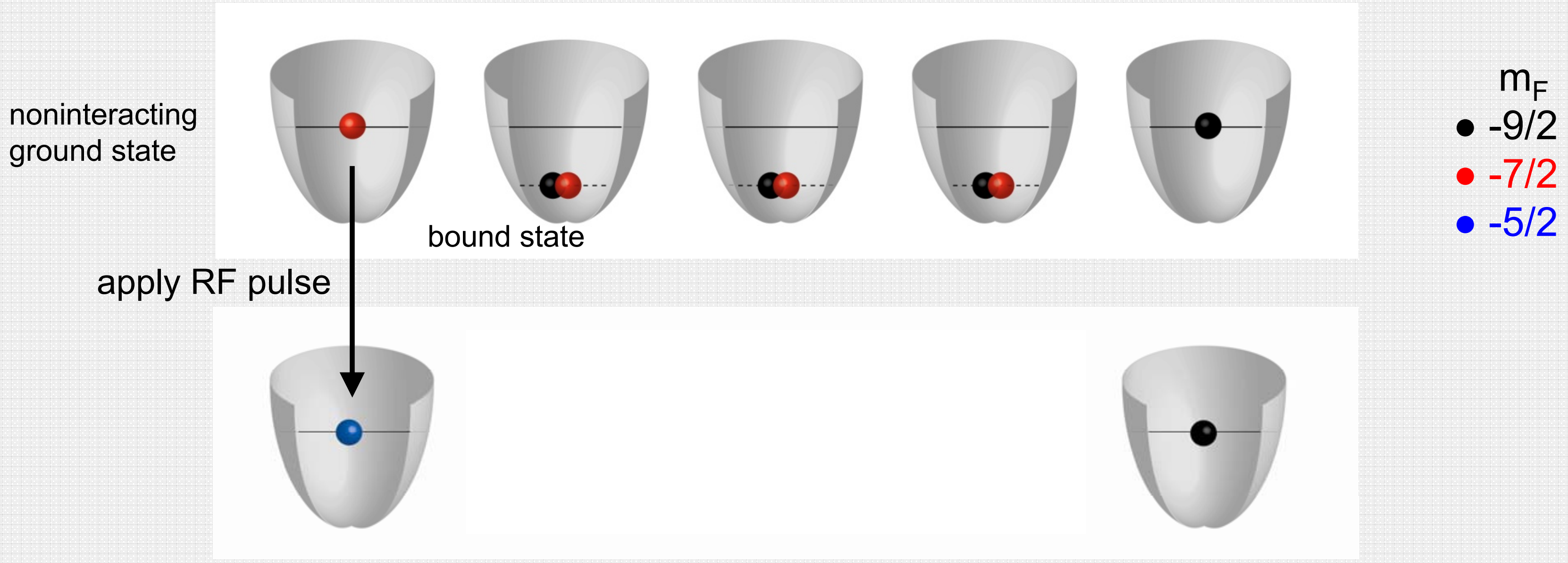
|9/2,-9/2> + |9/2,-5/2> mixture

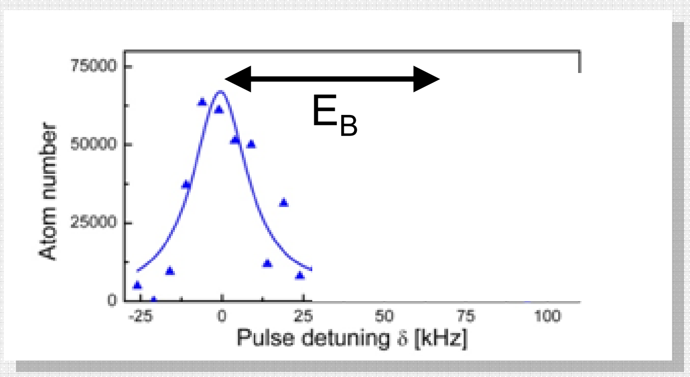
|9/2,-9/2> + |9/2,-7/2> mixture
(no Feshbach resonance)

Physics beyond the single band Hubbard model.

M. Köhl et al., Phys. Rev. Lett. 94, 080403 (2005)

Radio-frequency spectroscopy

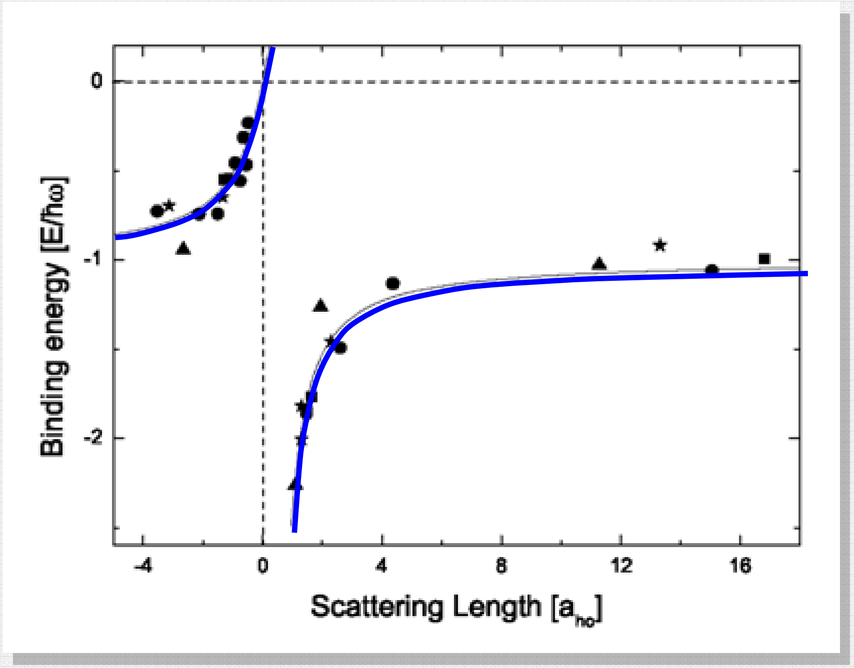




fraction of molecules measures on-site correlations $\langle n_{\uparrow} n_{\downarrow} \rangle$

Measuring the binding energy

Two particles in a harmonic oscillator: measuring the exact eigenstates



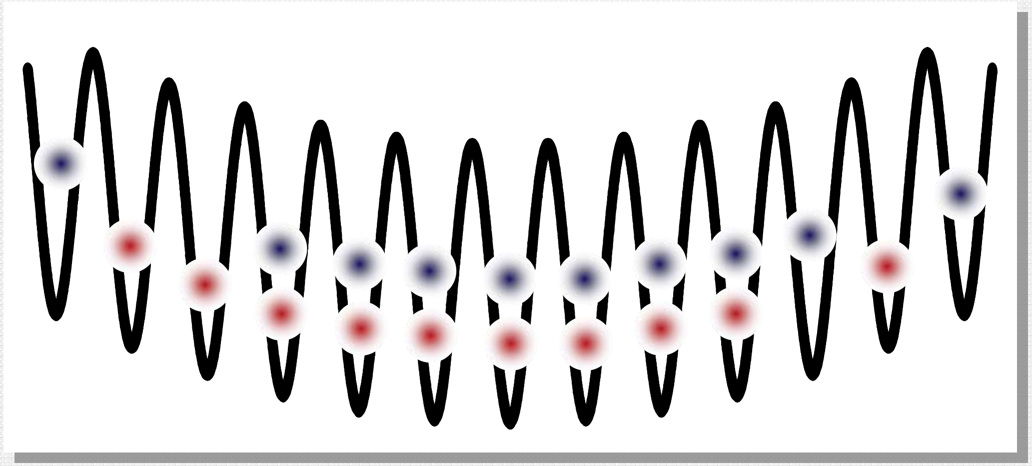
Theory according to
T. Busch et al., Found. Phys. 28, 549 (1998).

$$\frac{a_{ho}}{a} = \sqrt{2}\,\frac{\Gamma(-E_B/2\hbar\omega)}{\Gamma(-E_B/2\hbar\omega - 1/2)}$$

T. Stöferle, H. Moritz, K. Günter, M. Köhl, T. Esslinger, Phys. Rev. Lett. 96, 030401 (2006).

Thermometry in an optical lattice

Density of states



Deep lattice

$$E(x_i) = \frac{m}{2}\omega^2\left(\frac{\lambda i}{2}\right)^2$$

$$\rho(E) \propto E^{d/2-1} \qquad d: \text{dimensionality}$$

Same density of states as the gas in a box potential!

(for comparison: harmonic oscillator has $\rho(E) \propto E^{d-1}$)

Questions

1. loading the gas from the harmonic trap to the harmonic trap + lattice?
2. temperature dependence of number of holes in the Fermi sea?

Entropy

Entropy of a Fermi gas in a 3D power law potential $V(r) \propto r^{\alpha}$:

$$S = N\pi^2\left(\frac{1}{\alpha}+\frac{1}{2}\right)\frac{k_B T}{E_F} + O\left((T/T_F)^2\right)$$

Adiabatic loading into a lattice

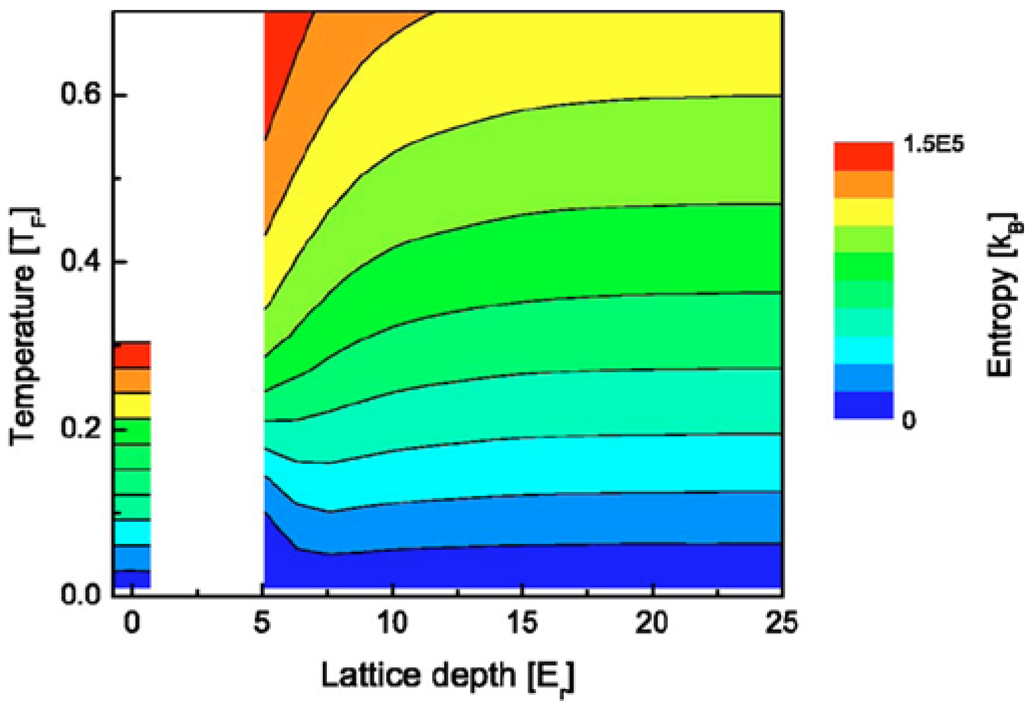
initial condition: harmonic trap (α=2) $S_{initial} = N\pi^2 \frac{k_B T}{E_F}$

final condition: "box" (α=∞) $S_{final} = \frac{N\pi^2}{2}\frac{k_B T}{E_F}$

Adiabatic heating of T/T_F by a factor of 2!

M. Köhl, Phys. Rev. A 73, 031601 (2006)

Change of entropy



M. Köhl, Phys. Rev. A 73, 031601 (2006)

A heating effect is also present for intermediate lattice depth and interacting systems.

The fraction of doubly occupied sites

molecules

$$n_2 = \frac{N_2}{N} = \frac{\int \rho(E) f^2(E) dE}{\int \rho(E) f(E) dE}$$

Fermi distribution

$$f(E) = \frac{1}{e^{(E-\mu)/kT} + 1}$$

Expand into power series around $E=\mu$ for low T (Sommerfeld expansion).

$$n_2 = \frac{N_2}{N} = 1 + \frac{1}{N}\sum_{n=0}^{\infty} a_{2n+1}(k_B T)^{2n+1} \left. \frac{d^{2n}\rho(\epsilon)}{d\epsilon^{2n}} \right|_{\epsilon=\mu}.$$

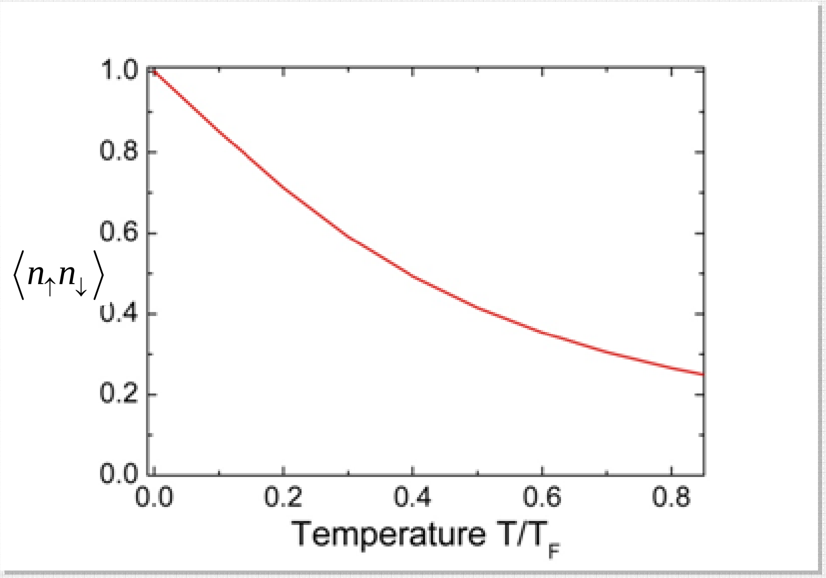
For a general density of states $\rho(E) \propto E^{\nu}$

$$n_2 = 1 - (\nu + 1)\frac{k_B T}{E_F} + \mathcal{O}\left[\left(\frac{k_B T}{E_F}\right)^2\right].$$

M. Köhl, Phys. Rev. A 73, 031601 (2006)

Measuring <$n_\uparrow n_\downarrow$> correlations

Temperature dependence in the band insulator



M. Köhl, Phys. Rev. A 73, 031601(R) (2006).

Summary

- Cold atoms and optical lattices
- Fermions in optical lattices, Fermi surface
- Tuning interactions by a Feshbach resonance
- Molecules in optical lattices
- Thermometry in an optical lattice