Cold Atoms and Optical Lattices Problems

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1. You were told that for one atom in a double well, there will always be an interference pattern in the ground state, for arbitrarily large barrier height (low coupling strength J).
    a. Show that this is not really true: In reality, the two sites are never exactly symmetric, but have a residual energy offset E between the left and the right side. Calculate the contrast (max-min)/(max+min) of the interference pattern as a function of this energy offset E.
    b. you showed in a) that there is no interference in reality when the coupling strength is reduced to very low levels. In real reality, however that does not have to be the case: If the atom is prepared in a superposition between left and right and the coupling is reduced non-adiabatically (and this is usually the case to some degree), then there can be interference without coupling. Assuming that the state does not change during the reduction of the coupling, and the residual offsets are randomly distributed (Gaussian distribution with RMS width R, in case you need to know) – what will happen and on which scale?

2. In analogy to the dynamics for a single atom in a double well, consider the case where two atoms are prepared both on the left side of a double well.
    a. Show that in this case, even if the interactions are neglected and the tunneling is therefore independent, the left-right density oscillation has not stayed exactly the same.
    b. Why did the behavior change anyway if the atoms don't interact and tunnel independently?
    c. Consider the case for very large interaction (U >> J), where interaction strongly influences the dynamics. Show that in this case, there is a sinusoidal oscillation again (in between it is not) and determine the frequency.
    d. Determine the resulting interference pattern. How strong is it? In free space, there is a connection between a phase gradient and motion of particles (momentum). Can you find the connection here between phase gradient (here the phase difference) and the velocity (flow) of density between the wells?

3. Many atoms in the double well: Make a mean field approximation to each of the wells (describe them in terms of density and phase) for large atom numbers and show that the symmetric system is modeled by the classical (that is, non-quantized) Josephson Hamiltonian describing a superconducting Josephson Junction in terms of macroscopic fields:
$$H = E_c d^2/8 - E_J\sqrt{N^2 - d^2} \cdot \cos(\varphi)$$
With E_c and E_J the Coulomb (interaction) and (tunnel) Junction energies per particle, respectively; φ and $d = (n_R - n_L)$ the phase difference and particle difference between left and right well and N the total particle number. What are the conditions to validly describe the double well in these terms?