Optical lattices with one, two or many atoms (2)

QUANTUM Mainz

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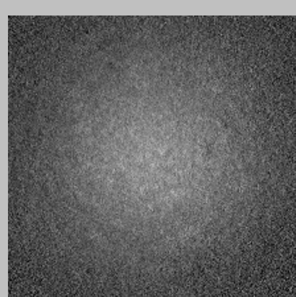
Simon Fölling, Oxford 2008

noise correlation measurement



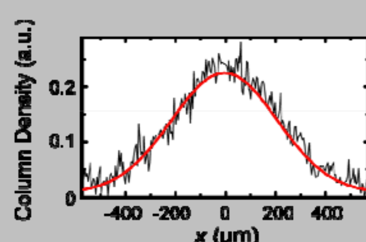
TOF image of fermionic band insulator after sudden release

Fermions: *no macroscopic occupation of single particle wave function*

↓

no interference peaks in density distribution

Bosons *in Mott insulator: no interference due to localization*



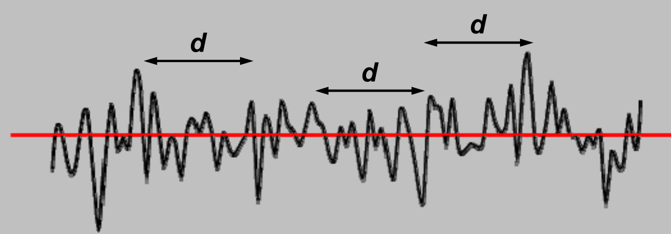
signature of the periodicity of the lattice?

envelope is determined by on-site single particle wave function (Wannier function)

granularity of matter wave: *atomic shot noise, atom number fluctuates*

$$\Delta N = \sqrt{N_{bin}}$$

are there correlations in the noise?



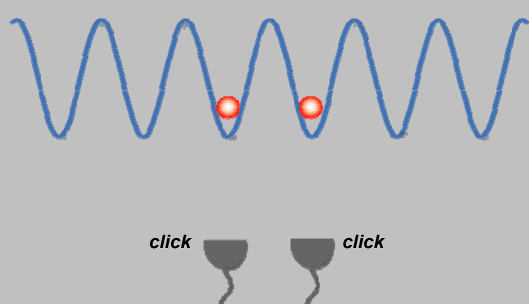
Hanbury-Brown Twiss measurement: are fluctuations correlated on special distances d?

$$g^{(2)}(\vec{d}\,) = \frac{\int \langle n(\vec{r}+\vec{d}/2)\cdot n(\vec{r}-\vec{d}/2)\rangle \; d^2r}{\int \langle n(\vec{r}+\vec{d}/2)\rangle\cdot\langle n(\vec{r}-\vec{d}/2)\rangle \; d^2r}$$

Quantitatively

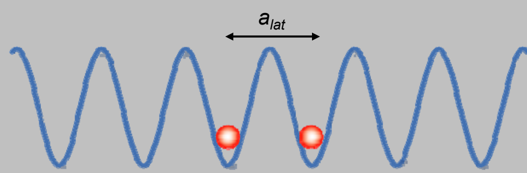
$g^{(2)}(d) > 1$	*noise correlated (Bosons)*
$g^{(2)}(d) = 1$	*noise uncorrelated*
$g^{(2)}(d) < 1$	*noise anti-correlated (Fermions)*

Hanbury Brown-Twiss effect for atoms



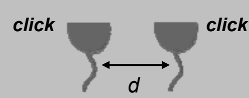
indistinguishable particles

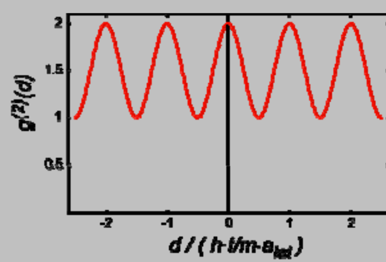
Hanbury Brown-Twiss effect for atoms



indistinguishable particles

there is another way …



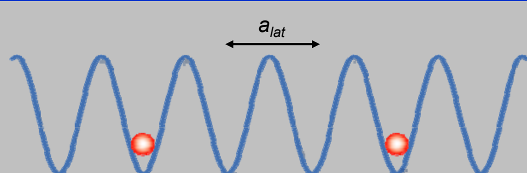


$$\left| \downarrow \quad \downarrow \pm e^{i\Phi(d)} \times \right|^2$$

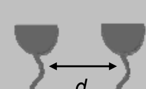
Φ: relative phase accumulated when propagating from source to detector

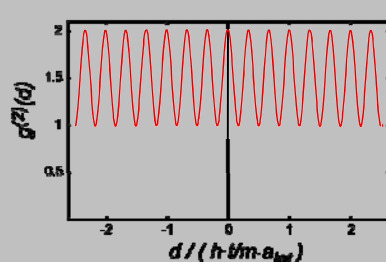
$$l = \frac{h}{m \cdot a_{lat}} t_{TOF}$$

Hanbury Brown-Twiss effect for atoms



indistinguishable particles





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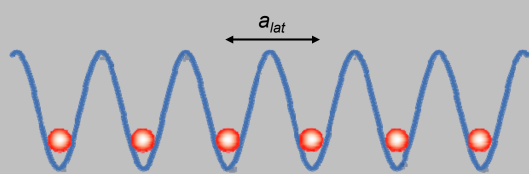
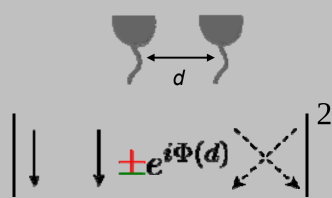
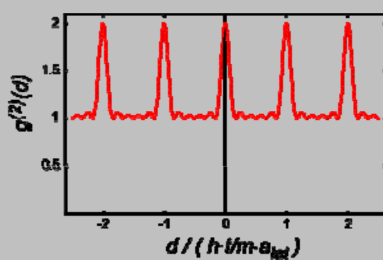
Hanbury Brown-Twiss effect for atoms



indistinguishable particles

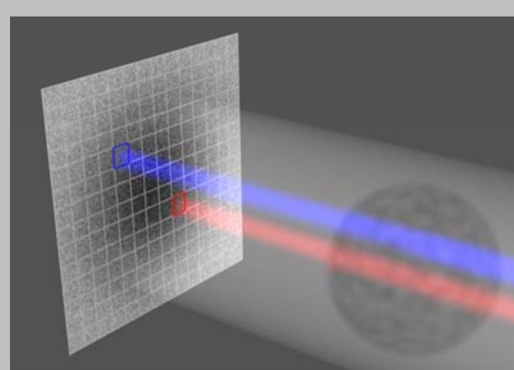




Φ: relative phase accumulated when propagating from source to detector

$$l = \frac{h}{m \cdot a_{lat}} t_{TOF}$$

Detectors for Atoms



Normalized Correlation Function

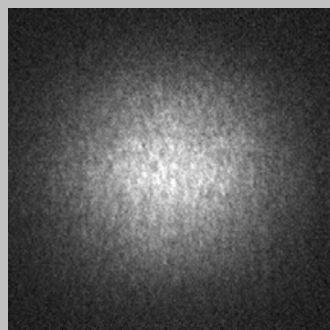
$$C(\vec{d}) = \frac{\int \langle n(\vec{x}) \cdot n(\vec{x}+\vec{d}) \rangle d^2\vec{x}}{\int \langle n(\vec{x}) \rangle \langle n(\vec{x}+\vec{d}) \rangle d^2\vec{x}} - 1$$

Corresponds to Second order Correlation

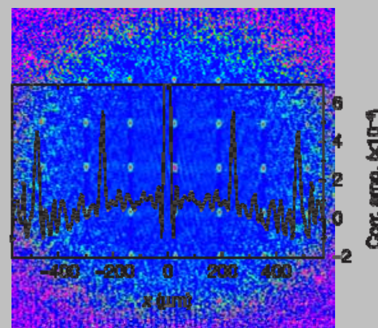
$$\langle \hat{n}(\mathbf{x}_1,t)\,\hat{n}(\mathbf{x}_2,t) \rangle = \langle \hat{a}^+(\mathbf{x}_1,t)\,\hat{a}(\mathbf{x}_1,t)\,\hat{a}^+(\mathbf{x}_2,t)\,\hat{a}(\mathbf{x}_2,t) \rangle$$

Hanbury Brown-Twiss effect for atoms

Bosonic ^{87}Rb



TOF image



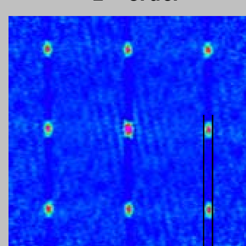
correlation function!

experiment: S. Fölling et al., Nature 434, 481 (2005)

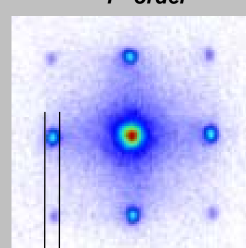
*theory: E. Altman, E. Demler & M. Lukin, PRA **70**, 013603 (2004).*

First order vs. second order correlation

2nd order



1st order



What about the sizes of the peaks?

Time of flight imaging from lattice

$$\langle \hat{n}(\boldsymbol{x}, t_{exp}) \rangle = \langle \hat{a}^\dagger(\boldsymbol{x}, t_{exp}) \hat{a}(\boldsymbol{x}, t_{exp}) \rangle = \left\langle \sum_j \hat{a}_j^\dagger(\boldsymbol{x}, t_{exp}) \sum_k \hat{a}_k(\boldsymbol{x}, t_{exp}) \right\rangle$$

Propagation from lattice Into free space:

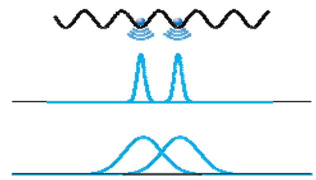
$$\hat{a}_j^\dagger(\boldsymbol{x}, t_{exp}) = W(\boldsymbol{x} - \boldsymbol{x}_j, t) e^{\frac{i\hbar t}{2m^2\sigma_0^2\sigma(t)^2}\boldsymbol{x}^2} \hat{a}_j^\dagger$$

Gaussian envelope:

$$W(\boldsymbol{x}, t) = \frac{2\pi^{3/4}}{\sqrt{\sigma_0 + \frac{i\hbar t}{m\sigma_0}}} e^{-\frac{\boldsymbol{x}^2}{2\sigma(t)^2}}$$

Width:

$$\sigma(t) = \sqrt{\sigma_0^2 + \frac{\hbar^2 t^2}{\sigma_0^2 m^2}}$$



Together:

$$n(\boldsymbol{x}, t_{exp}) = \sum_{j,k} W^*(\boldsymbol{x} - \boldsymbol{x}_j) W(\boldsymbol{x} - \boldsymbol{x}_k) e^{\frac{i\hbar t_{exp}}{2m^2\sigma_0^2\sigma(t_{exp})^2}(2\boldsymbol{x}(\boldsymbol{x}_j - \boldsymbol{x}_k) + \boldsymbol{x}_k^2 - \boldsymbol{x}_j^2)} \hat{a}_j^\dagger \hat{a}_k$$

First order correlation

$$\hat{a}^\dagger(\boldsymbol{x}, t_{exp}) \hat{a}(\boldsymbol{x}, t_{exp})$$

$$= \sum_{j,k} W^*(\boldsymbol{x} - \boldsymbol{x}_j) W(\boldsymbol{x} - \boldsymbol{x}_k) e^{\frac{i\hbar t_{exp}}{2m^2\sigma_0^2\sigma(t_{exp})^2}(2\boldsymbol{x}(\boldsymbol{x}_j - \boldsymbol{x}_k) + \boldsymbol{x}_k^2 - \boldsymbol{x}_j^2)} \hat{a}_j^\dagger \hat{a}_k$$

Assumption: $x_{j,k} << x$ can be neglected both in W(x) and in phase term

$$\approx |W(\boldsymbol{x})|^2 \sum_{j,k} e^{\frac{im\boldsymbol{x}}{\hbar t_{exp}}(\boldsymbol{x}_j - \boldsymbol{x}_k)} \left\langle \hat{a}_j^\dagger \hat{a}_k \right\rangle$$

As $x_{j,k}$ are evenly spaced by a, this is a Fourier sum (in 1D with $x_j - x_k = a(j-k)$)

Quantitative analysis: Gerbier et al., condmat:0808.2212

Second order correlation (1)

Second order correlation:

$$\hat{n}(x_1,t)\hat{n}(x_2,t) =$$

$$\sum_{j,k,l,m} W^*(x_1 - x_j)W(x_1 - x_k)W^*(x_2 - x_l)W(x_2 - x_m) \cdot$$

$$e^{\frac{i\hbar t_{exp}}{2m\sigma_0^2\sigma(t_{exp})^2}(2\boldsymbol{x}_1(\boldsymbol{x}_j - \boldsymbol{x}_k) + \boldsymbol{x}_k^2 - \boldsymbol{x}_j^2) + (2\boldsymbol{x}_2(\boldsymbol{x}_l - \boldsymbol{x}_m) + \boldsymbol{x}_m^2 - \boldsymbol{x}_l^2)} \hat{a}_j^\dagger \hat{a}_k \hat{a}_l^\dagger \hat{a}_m$$

Assumption: $x_{j,k}$ << x can be neglected only in W(x)

$$\approx \sum_{j,k,l,m} |W(x_1)|^2 |W(x_2)|^2 e^{\frac{im}{2\hbar t}(2\boldsymbol{x}_1(\boldsymbol{x}_j - \boldsymbol{x}_k) + \boldsymbol{x}_k^2 - \boldsymbol{x}_j^2) + (2\boldsymbol{x}_2(\boldsymbol{x}_l - \boldsymbol{x}_m) + \boldsymbol{x}_m^2 - \boldsymbol{x}_l^2)} \hat{a}_j^\dagger \hat{a}_k \hat{a}_l^\dagger \hat{a}_m$$

For the Mott insulator, we know g_1:

neglecting corrections for simplicity

$$\hat{a}_j^\dagger \hat{a}_k = \delta_{jk} n_j$$

$$\hat{a}_j^\dagger \hat{a}_k \hat{a}_l^\dagger \hat{a}_m = \hat{a}_j^\dagger(\hat{a}_l^\dagger \hat{a}_k + \delta_{lk})\hat{a}_m \approx \delta_{jk}\delta_{lm} n_j n_l + \delta_{jm}\delta_{lk} n_j n_l$$

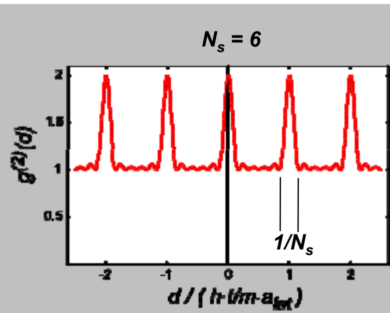
Second order correlation (2)

$$\hat{n}(x_1,t)\hat{n}(x_2,t) = \sum_{j,l} |W(x_1)|^2 |W(x_2)|^2 \cdot e^{\frac{im}{\hbar t}(\boldsymbol{x}_2 - \boldsymbol{x}_1)(\boldsymbol{x}_l - \boldsymbol{x}_j)} n_j n_l =$$

$$\sum_{j,l} |W(\bar{x} - \boldsymbol{d}/2)|^2 |W(\bar{x} + \boldsymbol{d}/2)|^2 \cdot e^{\frac{im}{\hbar t}(\boldsymbol{x}_l - \boldsymbol{x}_j)\cdot \boldsymbol{d}} n_j n_l$$

For a 1D String of N_s sites with spacing a

$$\sum_{k=1}^{N_s} \sum_{j=1}^{N_s} e^{i\frac{m}{\hbar t}dka} e^{-i\frac{m}{\hbar t}dja} = \frac{\sin(\pi N_s d/l)^2}{\sin(\pi d/l)^2}$$

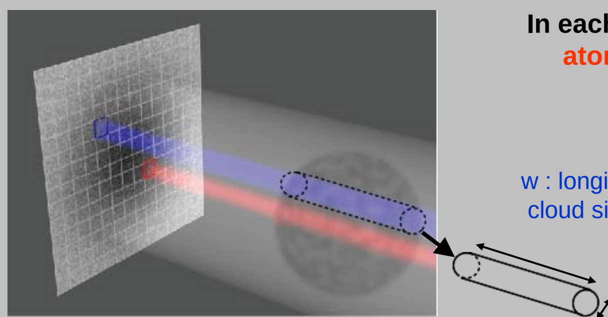


Detectors for Atoms

The atom cloud is only projected to a 2-Dimensional detector surface

Atom density is in fact integrated over a column parallel to the probe.

In each bin, $N_{bin} \gg 1$ atoms are counted.



w : longitudinal integration: cloud size

σ: transversal integration: imaging resolution

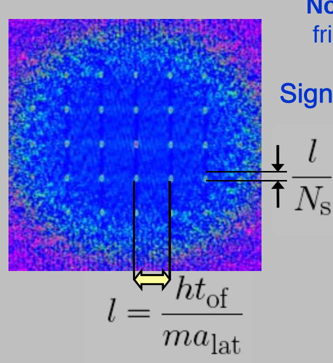
Amplitude of Measured Signal

2nd order coherence length : also ideal peak width $L_{\mathrm{coh}} \sim \frac{\hbar t}{m N_{\mathrm{s}} a_{\mathrm{lat}}} = \frac{l}{N_{\mathrm{s}}}$

Great spatial resolution :
fringe spacing l >> L_{coh} >> **res.** $\mathcal{C}_{\max} \approx 1 + 1$

Not so great spatial resolution :
fringe spacing l >> **res.** >> L_{coh} $\mathcal{C}_{\max} \approx 1 + \left(\frac{L_{\mathrm{coh}}}{\mathrm{res}}\right)^3$

Signal is reduced by the ratio of peak size to resolution



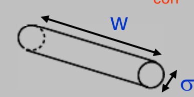
Theoretically! in reality:

Limited by resolution of imaging system, reducing the peak height (smoothing)

Amplitude of Correlation Signal

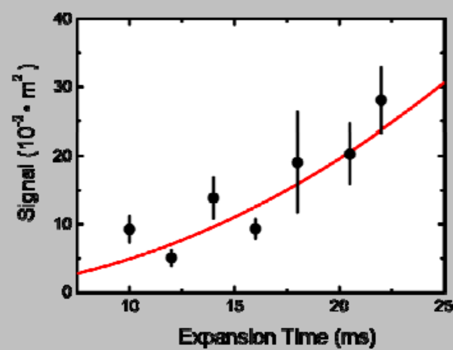
$$\mathcal{C}_{\max} \approx 1 + \frac{1}{N_s} \cdot \frac{1}{N_s^2}\left(\frac{l}{\sigma}\right)^2$$

Integration over „bin“ in Imaging plane: $l \gg \sigma > L_{coh}$

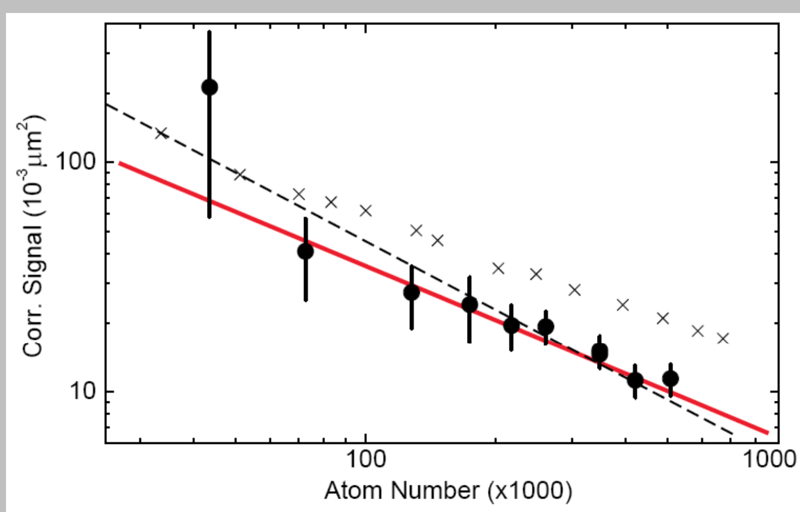


Integration in Probe direction : $w \gg l$

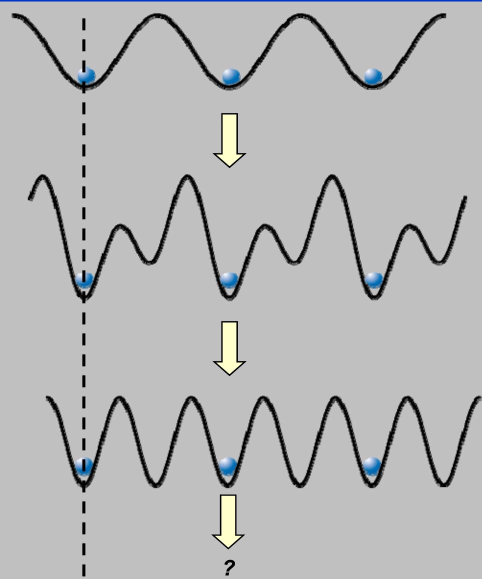
Changing l by changing the time of flight: quadratic dependency



Changing the size of the system



Patterned loading of the lattice



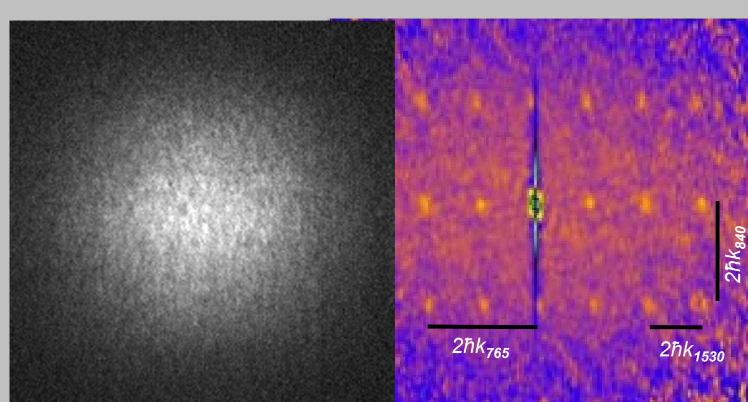
Mott State in long lattice

Add normal lattice

Switch off long lattice

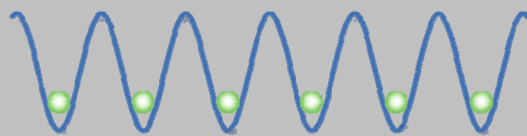
Release

Patterned correlations

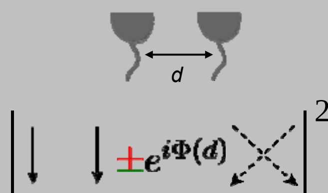


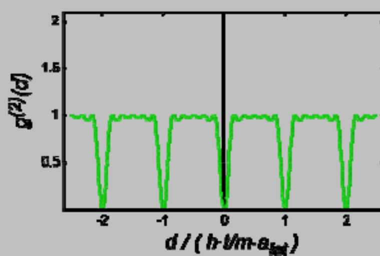
Patterned loading of split BEC: see S. Peil et. al., PRA 67, 051603 (2003)

Hanbury Brown-Twiss effect for atoms



indistinguishable particles

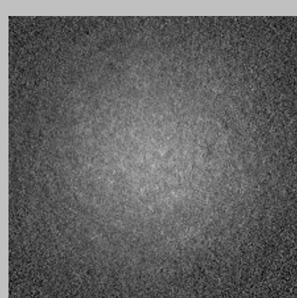




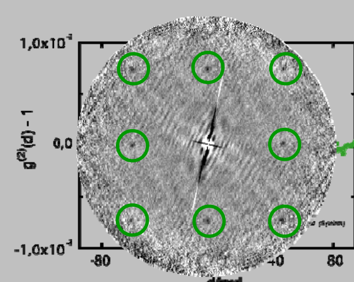
Φ: relative phase accumulated when propagating from source to detector

Hanbury Brown-Twiss effect for atoms

Fermionic ^{40}K



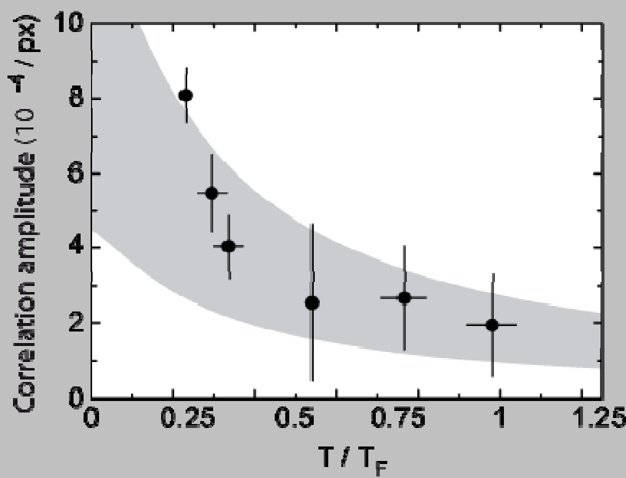
TOF image



correlation function!

*theory: E. Altman, E. Demler & M. Lukin, PRA **70**, 013603 (2004).*

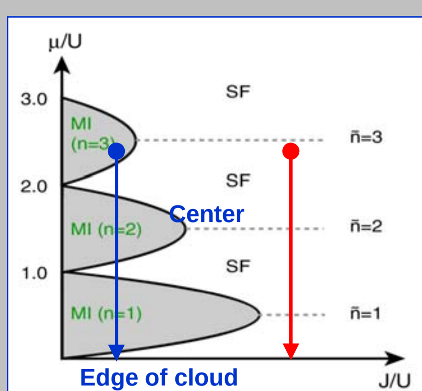
Change of local correlations due to temperature



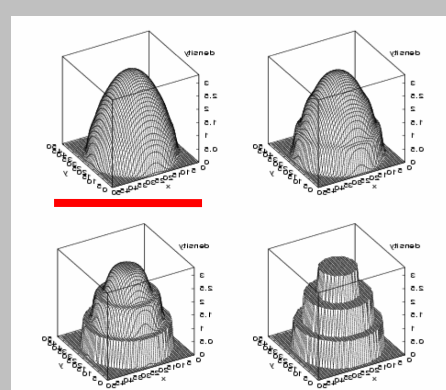
(before loading)

Shell structure of trapped Mott Insulator

Phase Diagram of homogeneus System



Density inhomogeneus System

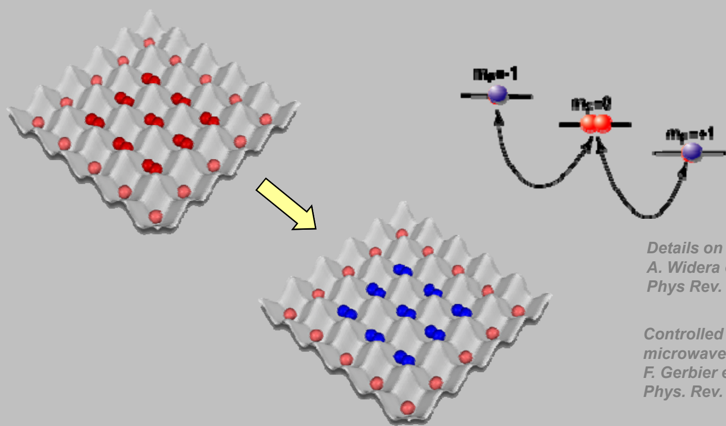


Figures courtesy of M. Niemeyer and H. Monien (Bonn)

*D. Jaksch et al. PRL **81**, 3108 (1998)*

Probing the on-site distribution

Spin-changing collisions transfer atom pairs to different spin state

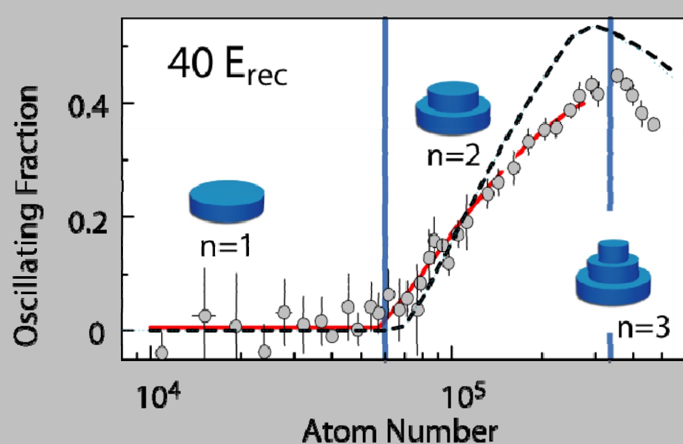


Details on spinor dynamics:
A. Widera et al.
Phys Rev. Lett. 95, 190405 (2005)

Controlled dynamics via microwave shifts:
F. Gerbier et al.
Phys. Rev. A 73, 041602(R) (2006)

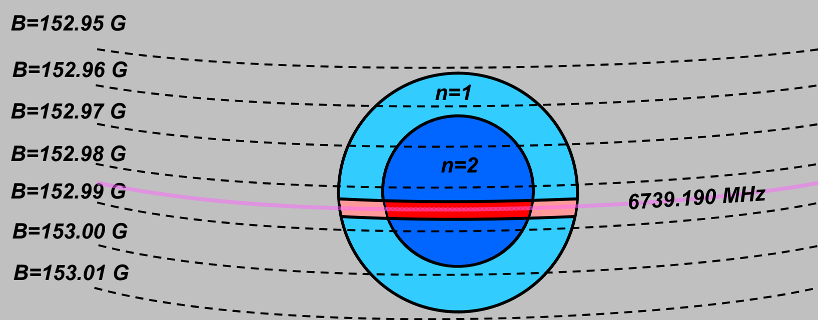
Using the interaction shifts using microwave spectroscopy:
G. Campbell et al., Science 313, 649 (2006)

Number squeezing in deep MI state

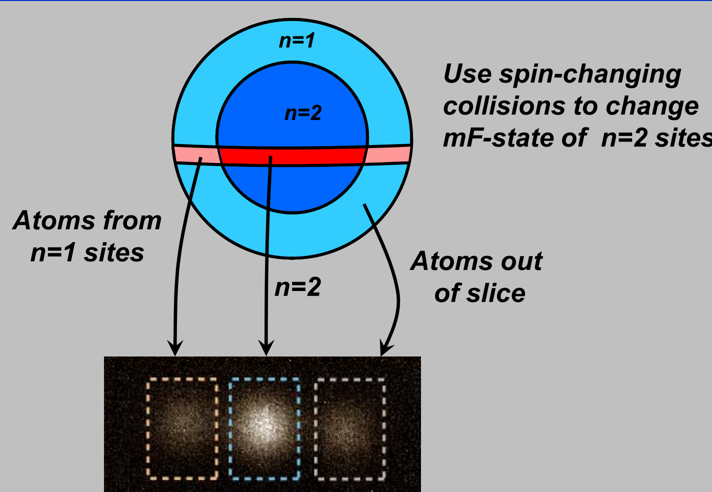


F. Gerbier at al., Phys Rev. Lett. 96, 090401 (2006)

Magnetic resonance tomography of the MI distribution

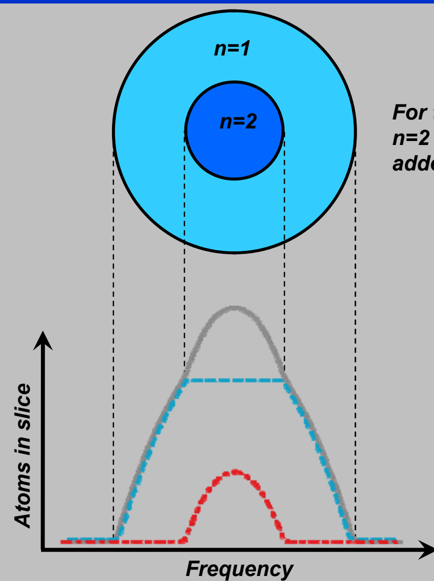


Magnetic resonance tomography of the MI distribution



Separate m_F-States by Stern-Gerlach field gradient

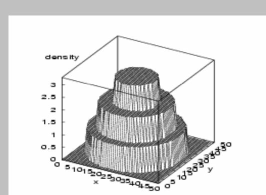
Profile of Double-Shell Mott Insulator

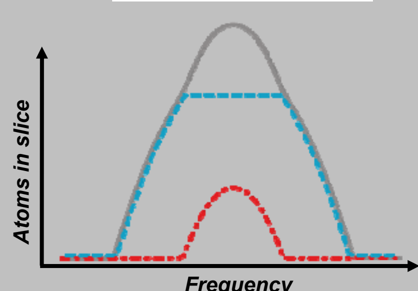


For the additional density in the n=2 shell, a second parabola is added on top of the n=1 shell

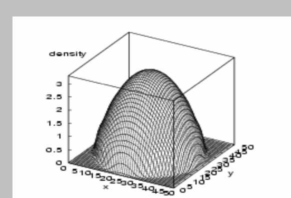
And the superfluid?

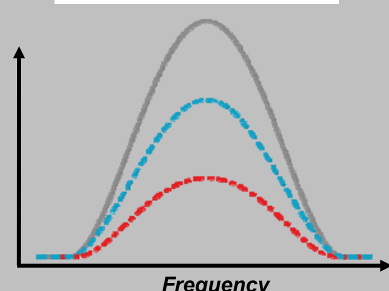
Mott shell structure



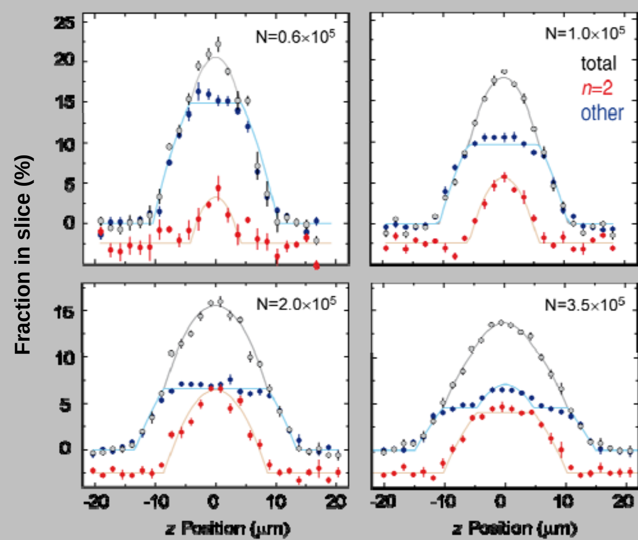


Superfluid: Thomas-Fermi distribution





Profiles of Mott insulator shells



Crossing the superfluid to Mott insulator transition