# Rotating Bose gases

- Connection with other rotating quantum systems

  *Rotating bucket experiment with liquid helium, neutrons stars, rotating nuclei, superconductor in a magnetic field*

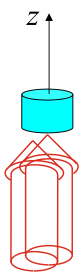
- Connection with quantum Hall physics

# 1.

# Bose gases in rotation

# and vortex nucleation

## Physics in a rotating frame




Cylindrically symmetric trap potential in the xy plane:

$$V(\vec{r}) = \frac{1}{2} m\omega^2 r^2 \qquad r^2 = x^2 + y^2$$

Stir at frequency $\Omega$ (with a rotating laser beam for example)

$$\delta V(\vec{r}, t) = \frac{\epsilon}{2} m\omega^2 \left(X^2 - Y^2\right) \qquad \epsilon \sim \text{a few } \%$$

**Hamiltonian in the rotating frame:**

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 r^2 - \Omega L_z$$
$$= \frac{\left(\vec{p} - \vec{A}\right)^2}{2m} + \frac{1}{2} m \left(\omega^2 - \Omega^2\right) r^2 \qquad \vec{A} = m\,\vec{\Omega} \times \vec{r}$$

Same physics as charged particles in a magnetic field + harmonic confinement

## Classical vs. quantum rotation

**Rotating classical gas**

velocity field of a rigid body $\vec{v} = \vec{\Omega} \times \vec{r}$ → $\vec{\nabla} \times \vec{v} = 2\vec{\Omega}$

**Rotating a quantum macroscopic object**

macroscopic wave function: $\psi(\vec{r}) = \sqrt{\rho(\vec{r})}\; e^{i\phi(\vec{r})}$

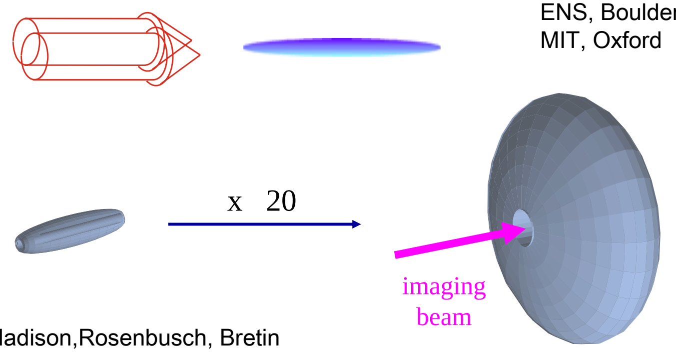
In a place where $\rho(\vec{r}) \neq 0$ , irrotational velocity field: $\vec{v} = \frac{\hbar}{m} \vec{\nabla}\phi$

The only possibility to generate a non-trivial rotating motion is to nucleate quantized vortices (points in 2D or lines in 3D)

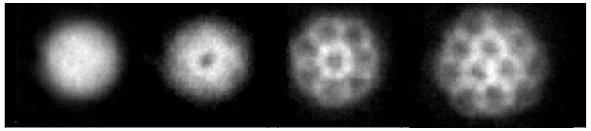
$$\oint \vec{v} \cdot \vec{dr} = \eta \frac{h}{m}$$

Feynman, Onsager, Pitaevskii

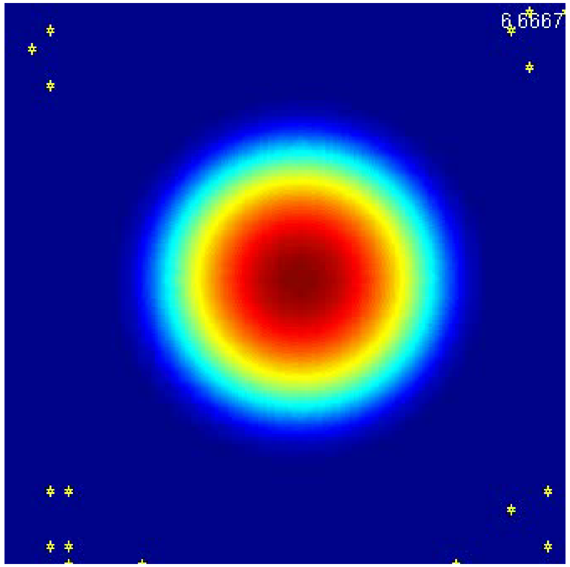
## Vortices in a stirred condensate




ENS 2000: Chevy,Madison,Rosenbusch, Bretin



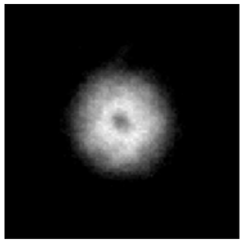

## Nucleation of vortices: a simulation using Gross-Pitaevskii equation

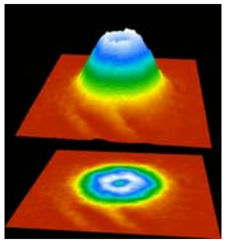



C. Lobo,
A. Sinatra,
Y. Castin,
Phys. Rev. Lett. **92**, 020403 (2004)

## The single vortex case

After time-of-flight expansion:





**Questions which have been answered:**

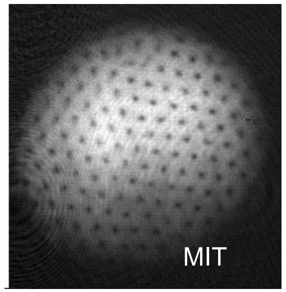
- Total angular momentum $N\hbar$ (i.e. $\hbar$ per particle) ?
- Is the phase pattern varying as $e^{i\theta}$ ?
- What is the shape of the vortex line?
- Can this line be excited (as a guitar string)? Kelvin mode

## The intermediate rotation regime

The number of vortices is notably larger than 1.

However one keeps the rotation frequency $\Omega$ notably below $\omega$

core size $\xi$ << vortex spacing




### What gives the vortex surface density ?

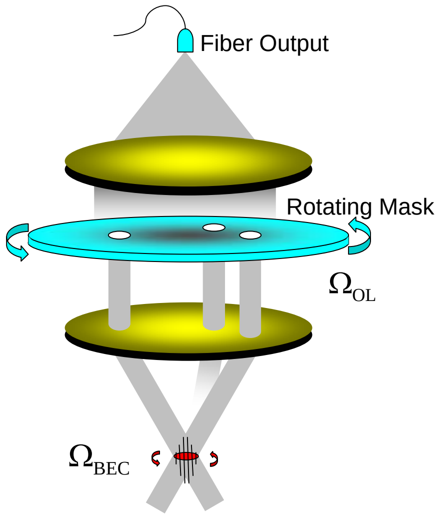
Uniform surface density of vortices $n_v$ with

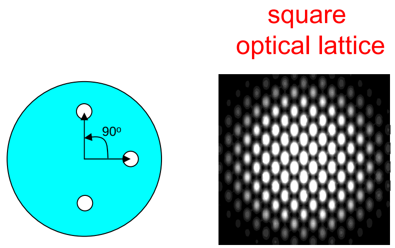
$$\Omega = \frac{\pi\hbar}{m} n_v$$

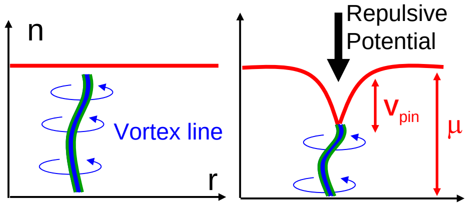
Coarse-grain average for the velocity field

$$\vec{v} = \vec{\Omega} \times \vec{r}$$

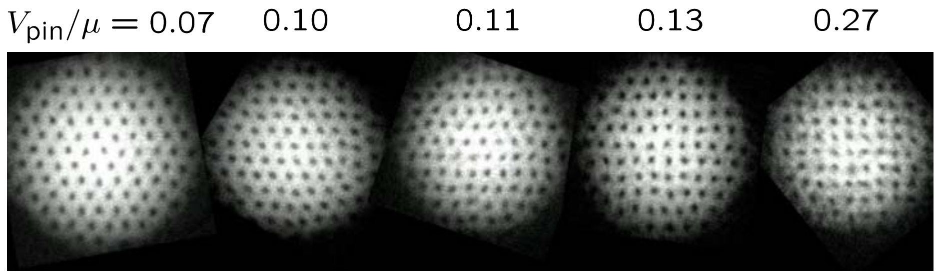
## Boulder: Rotating 2d optical lattice – Setup

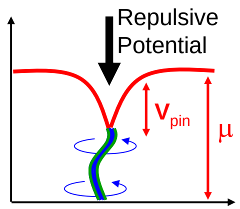



## Boulder: Pinning-induced structural phase transition




## 2.

## Are rotations compulsory to nucleate "steady-state" vortices?

No ! Geometric phases (like Berry's phase) can do the same job

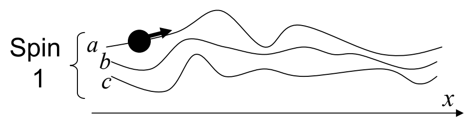
Olshanii-Dum, Tin-Lu Ho, Osterloh et al., Juzeliunas et al.,…

Our goal here: give a physical understanding of these "geometric" fields for Quantum Optics

M. Cheneau, S.P. Rath, T. Yefsah, K.J. Günter, G. Juzeliunas, J. Dalibard, Europhysics Letters **83**, 60001 (2008)

## How do geometrical potentials emerge?

Atom with several sub-levels that depend on space




$$i\hbar \frac{d}{dt}\begin{pmatrix}\psi_a\\ \psi_b\\ \psi_c\end{pmatrix} = \hat{H}\begin{pmatrix}\psi_a\\ \psi_b\\ \psi_c\end{pmatrix}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \sum_{j=a,b,c} E_j \hat{P}_j$$

The atom follows adiabatically the state $a$

Adiabatic elimination of the other states $b, c, \ldots$

$$i\hbar \frac{d\psi_a}{dt} = \hat{H}'\psi_a \qquad \hat{H}' = \frac{(\vec{p} - \vec{A})^2}{2m} + E_a + V$$

$$\vec{A}(\vec{r}) = i\hbar \langle a|\vec{\nabla} a\rangle$$

geometric vector potential

$$V(\vec{r}) = \frac{\hbar^2}{2m}\sum_{j\neq a} |\langle a|\vec{\nabla} j\rangle|^2$$

geometric scalar potential

## Rotation vs. geometric phases

**Rotation: Sagnac phase**

Hamiltonian in xy plane (rotating frame):

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 r^2 - \Omega L_z$$

$$= \frac{(\vec{P}-\vec{A})^2}{2m} + \frac{1}{2}m(\omega^2-\Omega^2)r^2$$

Vector potential: $\vec{A} = m\vec{\Omega}\times\vec{r}$

*Coriolis force*

Scalar potential: $-\frac{1}{2}m\Omega^2 r^2$

*Centrifugal force*

**Geometric: Berry's phase**

Hamiltonian in the lab frame:

$$\hat{H}' = \frac{(\vec{p}-\vec{A})^2}{2m} + E_a + V$$

Vector potential: $i\hbar\langle a|\vec{\nabla}a\rangle$

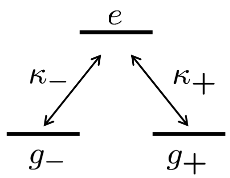
*"Lorentz" force ?*

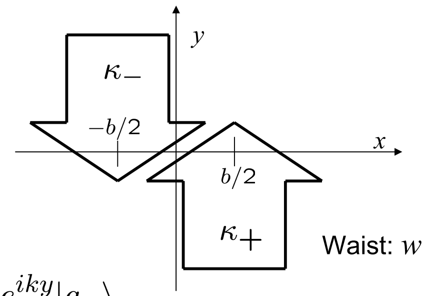
Scalar potential: $\frac{\hbar^2}{2m}\sum_{j\neq a}|\langle a|\vec{\nabla}j\rangle|^2$

*???*

## An implementation in Quantum Optics

Juzeliunas et al, 2006




Waist: $w$

$$|a\rangle \propto \kappa_- e^{-iky}|g_+\rangle - \kappa_+ e^{iky}|g_-\rangle$$

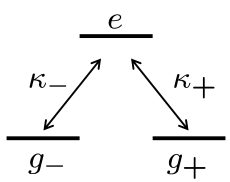
"Dark" or "non-coupled" state: $E_a = 0$

?

Scalar potential: $U(x) \simeq \frac{\hbar^2 k^2}{2M}G(x)$  $G(x) = \cosh^{-2}(2xb/w^2)$

Vector potential: $\longrightarrow$ Effective magnetic field: $\vec{B}(x) = \frac{2\hbar k b}{w^2}G(x)\vec{u}_z$; Unit charge: $q = 1$

## Physical origin of the scalar potential




$$|a\rangle \propto \kappa_- e^{-iky}|g_+\rangle - \kappa_+ e^{iky}|g_-\rangle$$

Atom-laser coupling:

$$V_{\text{AL}} = \kappa_+ e^{iky}\,|e\rangle\langle g_+| + \kappa_- e^{-iky}\,|e\rangle\langle g_-| + \text{h.c.}$$

The state $|a\rangle$ is an eigenstate of $V_{AL}$ with eigenvalue 0

But $|a\rangle$ is not an eigenstate of the force operator: $\vec{F}_{\text{AL}} = -\vec{\nabla}V_{\text{AL}}$

More precisely: $\langle a|\vec{F}_{\text{AL}}|a\rangle = 0$ but $\langle a|(\vec{F}_{\text{AL}})^2|a\rangle \neq 0$

The quantum fluctuations of the force create a fast micro-motion of the atom.

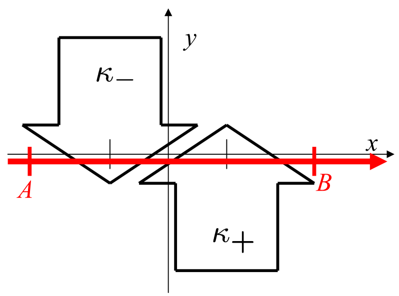
The kinetic energy of this micro-motion plays the role of a potential energy for the slow center-of-mass motion ➡ Scalar potential

*Similar to the trapping potential in a Paul trap for ions or electrons*

## Origin of the Lorentz force

The atom moving along the $x$ axis with velocity $v_x$ feels the force:

$$F_y = v_x B$$




Momentum kick:

$$\Delta p_y = \int_{t_A}^{t_B} v_x\, B(x(t))\,dt = \int_A^B B(x)\,dx \quad \approx 2\hbar k$$

The atom follows adiabatically the state $|a\rangle \propto \kappa_-|g_+\rangle - \kappa_+|g_-\rangle$

At point A: $\kappa_- \gg \kappa_+ \longrightarrow |a\rangle \simeq |g_+\rangle$

At point B: $\kappa_- \ll \kappa_+ \longrightarrow |a\rangle \simeq |g_-\rangle$

The passage from $|g_+\rangle$ to $|g_-\rangle$ is caused by an absorption-stimulated emission process

*Momentum transfer:* $2\hbar k$

## 3.

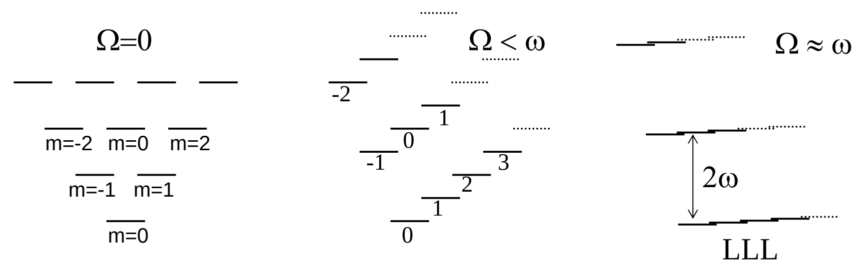
## The fast rotation regime:

## Physics in the Lowest Landau Level

## Landau levels for a rotating gas

Isotropic harmonic trapping in the $xy$ plane with frequency $\omega$

Hamiltonian in the rotating frame: $H - \Omega L_z$

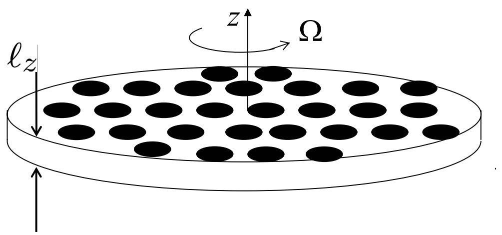



When $\Omega = \omega$, macroscopically degenerate ground state for the one-body hamiltonian (Rohshar, Ho)

$$H = \frac{\left(\vec{p} - \vec{A}\right)^2}{2m} \qquad \vec{A} = \vec{\Omega} \times \vec{r}$$

## Reaching the lowest Landau level

Assume that the $z$ direction is frozen, with a extension $\ell_z$

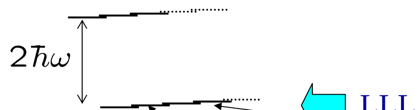



If the chemical potential and the temperature are much smaller than $2\hbar\omega$ , the physics is restricted to the lowest Landau level

This LLL regime corresponds to $1 - \frac{\ell_z}{Na} < \frac{\Omega}{\omega}$ $a$: scattering length

Typically: $N$=10 000 atoms, $\ell_z$=0.5 µm, $a$ =5 nm ⇒ $\Omega > 0.99\ \omega$

## The lowest Landau level




Characteristic length scale

$$a_0 = \sqrt{\hbar/m\omega}$$

$e^{-r^2/2a_0^2}$ $(x+iy)e^{-r^2/2a_0^2}$ $(x+iy)^n e^{-r^2/2a_0^2}$

General one-particle state in the LLL:

$$e^{-r^2/2a_0^2}\ P(x+iy) \longrightarrow e^{-r^2/2a_0^2} \prod_{j=1}^{n} (u - u_j)$$

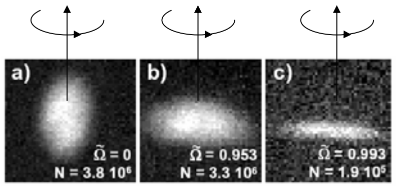
Polynomial or analytic function

$u = x + iy$ $u_j$ : vortices

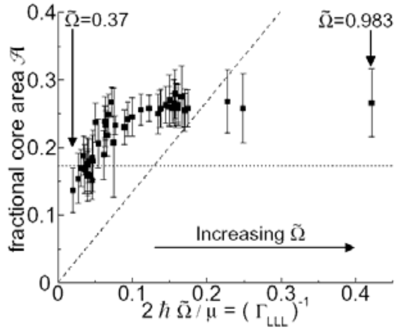
- The size of the vortices is comparable to their spacing
- The atom distribution is entirely determined from the vortex position

## Reaching the LLL experimentally

Evaporative spin-up method (Boulder): first rotate the gas at a moderate frequency and then evaporate of atoms with small angular momentum.




side view of the rotating BEC



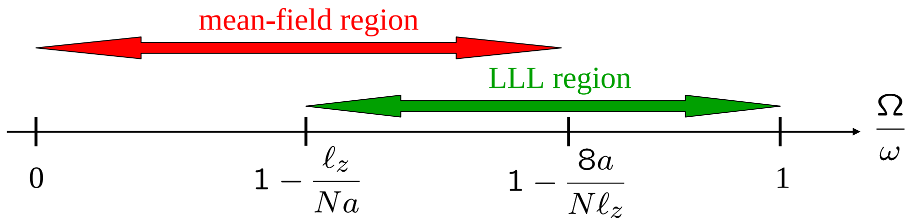

V. Schweikhard *et al.*, PRL **92**, 040404 (2004)

## Limits of the mean field approach

When $\Omega$ increases even more than the LLL threshold, the number of vortices $N_v$ increases and becomes similar to the atom number $N$

It occurs for $\frac{\Omega}{\omega} \geq 1 - \frac{8a}{N\ell_z}$

Total angular momentum: $L_z \sim N^2\hbar$ ($N\hbar$ per particle)




Note that $a < \ell_z$ for a 3D description of the binary collisions

## Beyond mean field: Laughlin states and more

For some specific filling factors $\nu = N/N_v$ the ground state is separated from the excited states by an energy gap

*incompressible states, analogous to those of Quantum Hall effect*

Example: $\nu = 1/2 \quad L = N(N-1)$

$$\Psi(\vec{r}_1, \ldots, \vec{r}_N) = \left[\prod_{j<k} (u_j - u_k)^2\right] e^{-\sum_j r_j^2 / 2a_0^2} \qquad u_j = x_j + iy_j$$

Eigenstate of the 1-body Hamiltonian in the LLL

Eigenstate of the interaction Hamiltonian: $V = g \sum_{j<k} \delta(\vec{r}_j - \vec{r}_k)$

## Conclusions

Low dimensional systems are not simpler than 3D systems!

➡ Several questions are still open concerning the static case, in particular in connection with superfluidity

➡ Concerning the rotating case, the analogy with the fractional quantum Hall effect has not yet been explored experimentally.

It is an experimental challenge to achieve vortex numbers similar to atom numbers, but the outcomes would be remarkable