§4. The Berezinskii-Kosterlitz-Thouless mechanism

A vortex in a Bose gas

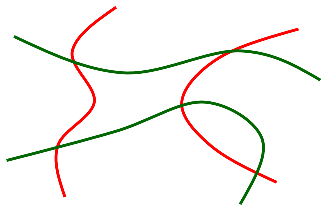
Consider a gas described by the macroscopic wave function

$$\psi(\vec{r}) = \sqrt{n(\vec{r})}\; e^{i\theta(\vec{r})}$$

A vortex is a point (or a line in 3D) where the density vanishes and around which the phase varies by $\eta \times 2\pi$ (η non-zero integer)

η is the "charge" of the vortex

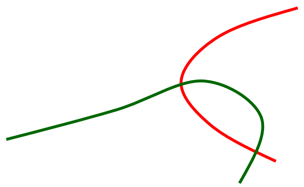
Vortices with charge $\eta = \pm 1$ appear naturally in a gas with a fluctuating wave function



Line along which $\mathcal{R}e(\psi) = 0$

Line along which $\mathcal{I}m(\psi) = 0$

How can vortices appear and disappear ?



Line along which $\mathcal{R}e(\psi) = 0$

Line along which $\mathcal{I}m(\psi) = 0$

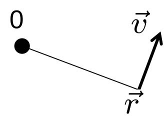
Annihilation/creation of two vortices with opposite charge

Energy of a vortex

The velocity field associated with a wave function is proportional to the gradient of the wave function

$$\vec{v}(\vec{r}) = \frac{\hbar}{m}\,\vec{\nabla}\theta(\vec{r}) \qquad \oint \vec{v}\cdot d\vec{r} = \pm\,\frac{2\pi\hbar}{m}$$

Single vortex in 2D



$$v = \pm\frac{\hbar}{mr}$$

Kinetic energy: $E_K = \frac{m}{2}\int n\; v^2\; d^2r \;\; = \frac{mn_0}{2}\,\frac{\hbar^2}{m^2}\int \frac{1}{r^2}\, d^2r$

Lower bound: size of the vortex core ξ Upper bound: size R of the gas

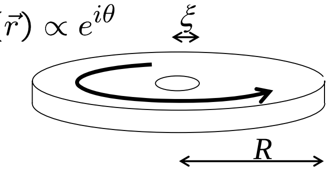
$$E_K = \frac{\pi\hbar^2 n_0}{m}\,\ln(R/\xi)$$

The KT mechanism for pedestrians

Probability for a vortex to appear as a thermal excitation?

One has to calculate the vortex free energy *E-TS* and compare it with *kT*

$\psi(\vec{r}) \propto e^{i\theta}$



$$v = \frac{\hbar}{mr} \qquad \rho_s = \frac{N}{\pi R^2}$$

Energy: $$E_K = \frac{\pi \hbar^2 n_0}{m} \ln(R/\xi)$$

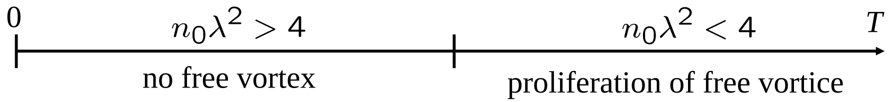
Entropy: $$S = k \log(W) \sim k \ln(R^2/\xi^2) = 2k \log(R/\xi)$$

Free energy of a vortex: $$\frac{E-TS}{kT} \sim \frac{1}{2}\left(n_0\lambda^2 - 4\right) \ln(R/\xi)$$

BKT transition for pedestrians (2)

Free energy of a vortex: $$\frac{E-TS}{kT} \sim \frac{1}{2}\left(n_0\lambda^2 - 4\right) \ln(R/\xi)$$

Thermodynamic limit $R \rightarrow +\infty$



When does the Kosterlitz-Thouless transition occurs?

The result $n_0\lambda^2 = 4$ is only a partial answer, because n_0 depends on temperature and it is usually unknown.

Analytical calculation by Fisher & Hohenberg, Monte-Carlo by Prokof'ev et al

$$n_{\text{total}}\lambda^2 = \ln\left(\frac{C}{\tilde{g}}\right) \qquad C = 380 \pm 3$$

For our setup with Rb cold atoms:

$$\tilde{g} = 0.13 \quad \Rightarrow \quad n_{\text{total}}\lambda^2 \simeq 8.0$$

This is not "universal" by contrast to the 3D result $n_{\text{total}}\lambda^3 \simeq 2.6$

What happens for a harmonically trapped interacting gas?

	Uniform, infinite	Harmonic trap (or other finite size system)
Ideal	No BEC	BEC $N > N_c$
Interacting	superfluid $n\lambda^2 > D_c$	?

Can it be a pure BEC transition?

No: in harmonic 2D, ideal BEC occurs when $n(0)\lambda^2 = \infty$

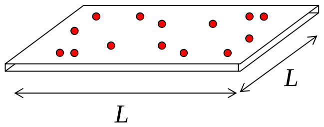
Can it be a pure superfluid transition with no long-range order?

No: in a finite size system, there is always a finite condensed fraction

The 'state of Texas' argument

For a finite size system in the region with quasi-long range order

$$g_1(r) \propto r^{-\alpha}$$



the condensed fraction is $f_0 \sim \left(\frac{\xi}{L}\right)^{\alpha}$

ξ : healing length (0.1 microns)

L : size of the sample (10 to 100 microns)

At the critical point: $\alpha = 1/4$ ➡ $f_0 \sim 0.2 - 0.3$

"With a magnetization < 0.01 as a reasonable estimate for the thermodynamic limit, the system would have to be bigger than the state of Texas for the Mermin-Wagner theorem to be relevant"

Bramwell – Holdsworth, 1994

§5. Experimental studies of 2D Bose gases

The MIT 2002 experiment

VOLUME 87, NUMBER 13 PHYSICAL REVIEW LETTERS 24 SEPTEMBER 2001

Realization of Bose-Einstein Condensates in Lower Dimensions

A. Görlitz,* J. M. Vogels, A. E. Leanhardt, C. Raman, T. L. Gustavson, J. R. Abo-Shaeer, A. P. Chikkatur, S. Gupta, S. Inouye, T. Rosenband, and W. Ketterle

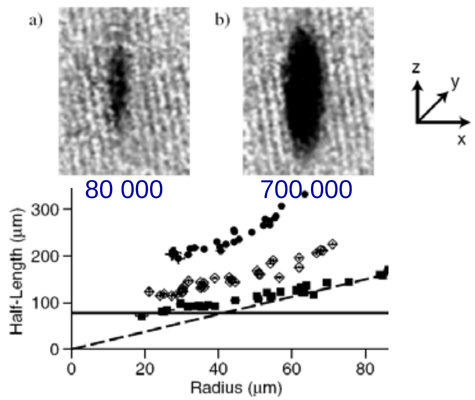
Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 23 April 2001; published 4 September 2001)

1064 nm laser focused with cylindrical lenses

frequency along z: 1000 Hz

frequencies in the xy plane: 30 and 10 Hz



The Innsbruck 2004 experiment

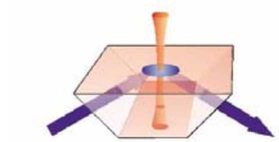
VOLUME 92, NUMBER 17 PHYSICAL REVIEW LETTERS week ending 30 APRIL 2004

Two-Dimensional Bose-Einstein Condensate in an Optical Surface Trap

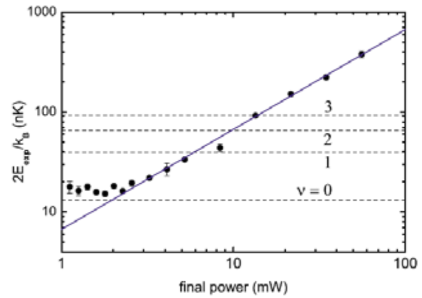
D. Rychtarik, B. Engeser, H.-C. Nägerl, and R. Grimm

Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria

(Received 23 September 2003; published 27 April 2004)



10 - 10 - 500 Hz

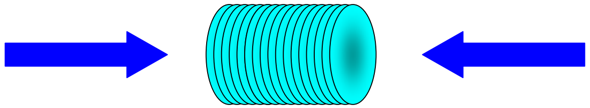


Expansion data in good agreement with the prediction derived from a Thomas-Fermi approximation.

Other 2D experiments

Oxford 2005: Hermite-Gaussian laser beam +magnetic trap

1D optical lattice on a 3D BEC (Yale, Florence, Zurich, Heidelberg,…)



optical lattice beams

Each site easily 2D

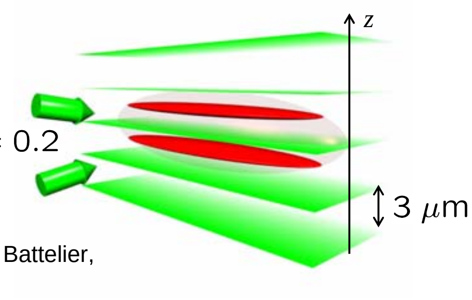
Need to suppress tunneling + hard to study a single plane
(not individually addressable + few atoms/site)

ENS experiment 2006: large period lattices

2 main sites with
a few 10^4 atoms in each

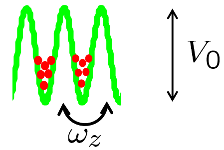
frequencies in the x-y plane:
10 - 100 Hz

$\theta = 0.2$



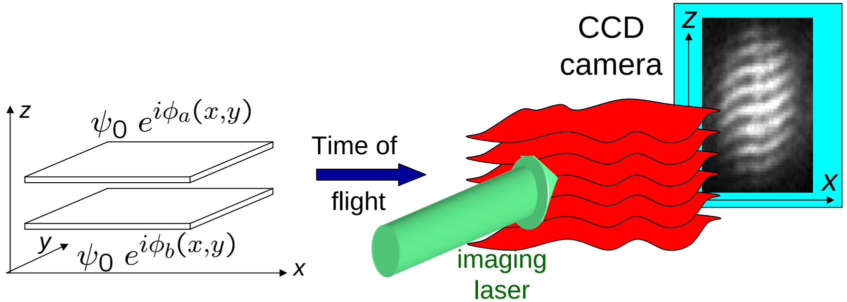
Z. Hadzibabic, P. Krüger, M. Cheneau, B. Battelier, and J. Dalibard, Nature 441, 1118 (2006)

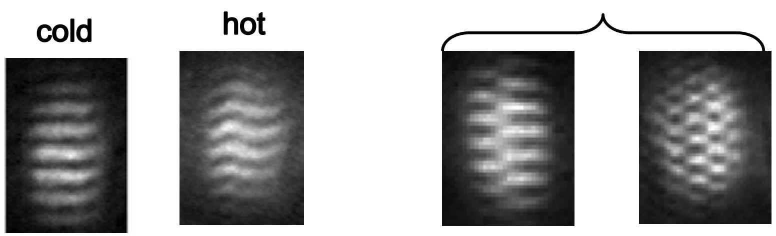
- ➡ Negligible tunnelling across the lattice barriers
- ➡ Adjustable temperature
- ➡ The quasi-condensate is in the 2D regime:



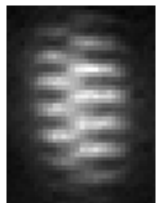
$$\mu \sim 2 \text{ kHz} \quad < \quad \hbar\omega_z \sim 4 \text{ kHz} \quad \ll \quad V_0 \sim 50 \text{ kHz}$$

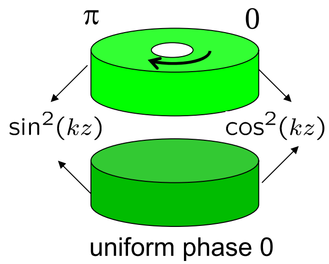
How to characterize the phase pattern of a 2D Bose gas





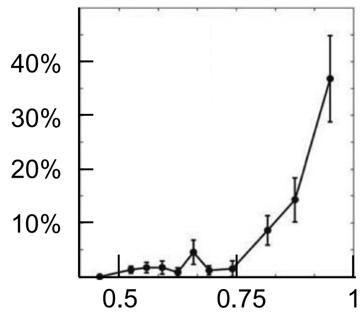
Dislocation = evidence for free vortices





Similar results at NIST

fraction of images showing a dislocation



temperature control (arb.units)

0 = full contrast at center

1 = no contrast at center

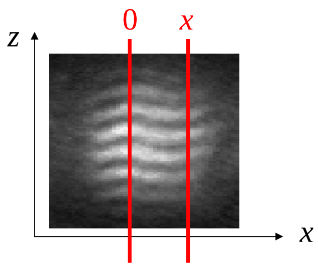
Investigation of the long range order: $g_1(\vec{r}) = \langle \psi^*(\vec{r})\, \psi(0) \rangle$

The interference signal between $\psi_a(x,y)$ and $\psi_b(x,y)$ gives

$$|\psi_a|^2 + |\psi_b|^2 + \psi_a^*\psi_b\, e^{ikz} + \psi_a\psi_b^*\, e^{-ikz}$$

$$\psi_a^*\psi_b \rightarrow \kappa(x,y)$$

$$\langle \kappa(x,y)\kappa^*(0)\rangle = \langle \psi_a^*(x,y)\psi_b(x,y)\psi_a(0)\psi_b^*(0)\rangle$$
$$= \langle \psi_a^*(x,y)\psi_a(0)\rangle\ \langle \psi_b(x,y)\psi_b^*(0)\rangle = |g_1(x,y)|^2$$



Altman-Demler-Polkovnikov

Good agreement with the predicted transition from algebraic to exponential decay, but:

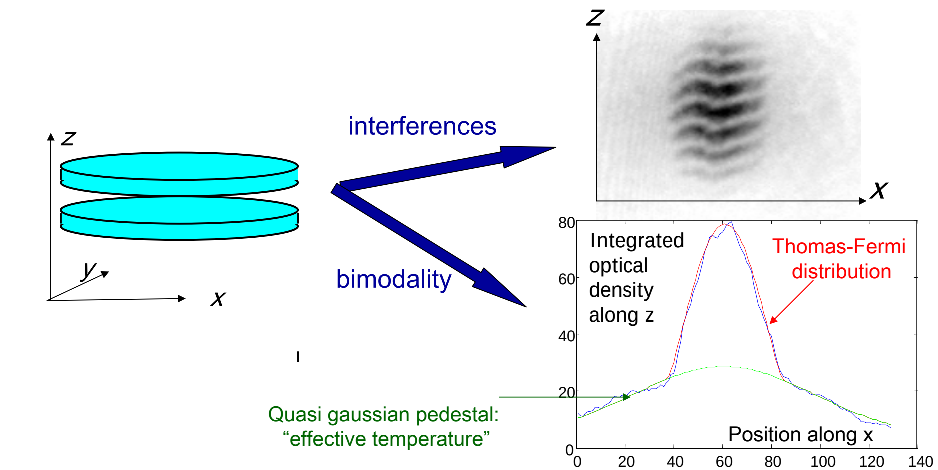
- Integration along the line of sight
- Non uniform system

§6. The critical point in a 2D Bose gas

Theory: Shlyapnikov-Gangardt-Petrov, Holtzman *et al.*, Kagan *et al.*, Stoof *et al.*, Mullin *et al.*, Simula-Blackie, Hutchinson *et al.*
Polkovnikov-Altman-Demler

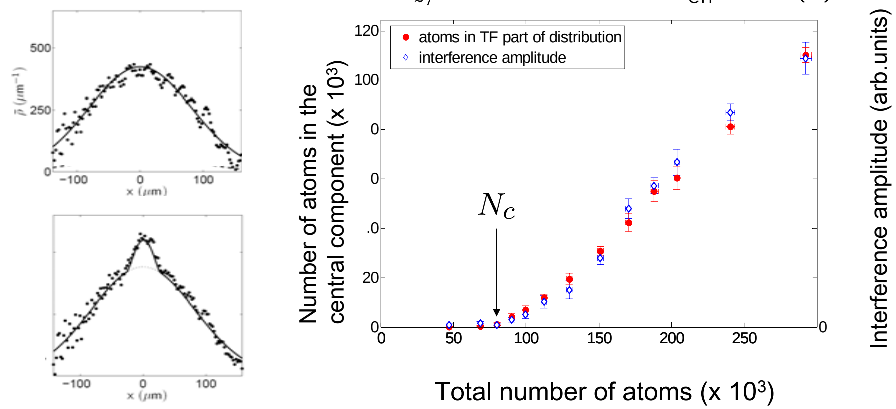
The critical point in a rubidium 2D gas

- For large temperatures, quasi-gaussian distributions, no interference
- For temperatures lower than a critical value:



Bimodality and interferences

$\hbar\omega_z/k = 150\,\mathrm{nK}$ $\qquad T_{\mathrm{eff}} = 92\,(6)\,\mathrm{nK}$



Within our accuracy, onset of bimodality and interference agree

Krüger, Hadzibabic, Dalibard, Phys. Rev. Lett. 99, 040402 (2007)

What is the nature of the critical point?

A simple hypothesis (Local Density Approximation): two-phase model

Normal fluid + superfluid

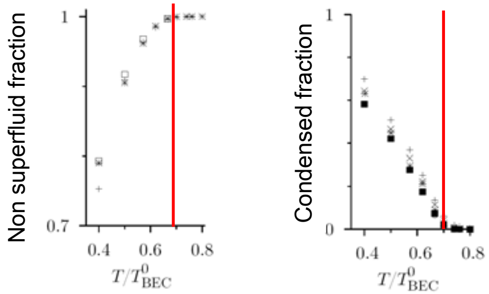
The critical point corresponds to the situation where the superfluid Kosterlitz-Thouless transition occurs at the center of the trap

Confirmed by a quantum Monte-Carlo analysis:

Holzmann-Krauth
PRL 100, 190402 (2008)

Similar conditions to the ENS experiment

Includes residual excitation of the 3rd dimension



Within this simple hypothesis, what is the corresponding total atom number ?

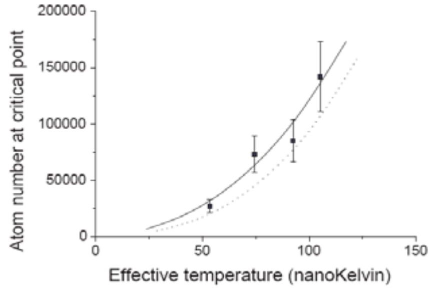
An hybrid 3D mean-field treatment

Hadzibabic, Kruger, Chenau, Rath, Dalibard, New J. Phys. 10 (2008) 045006.

Self consistent 3D Hartree-Fock analysis

$$V_{\mathrm{eff}}(\vec{r}) = V_{\mathrm{trap}}(\vec{r}) + 2g^{(3D)}n^{(3D)}(\vec{r})$$

- The z degree of freedom is treated in a fully quantum way (5 vibrational levels are included)
- The xy degrees of freedom are treated semi-classically (as usual)



Effective temperature deduced from a gaussian fit of the wings of the distribution

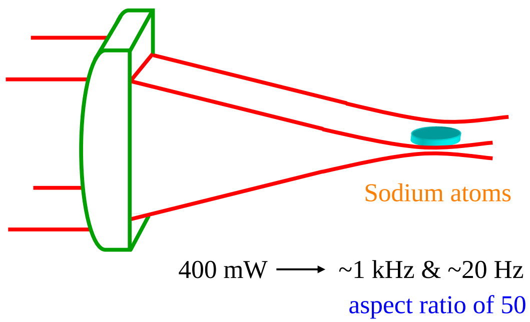
$$T_{\mathrm{eff}} \simeq 0.6 - 0.7\ T_{\mathrm{real}}$$

No adjustable parameters

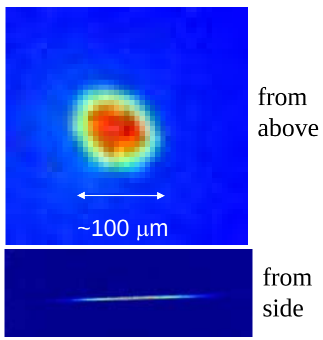
See also Holzmann, Chevalier, Krauth, *EPL* **82**, 30001 (2008)
Bisset, Blackie, arXiv:0809.0747

Study of a single 2D Bose gas at NIST

Cladé, Ryu, Ramanathan, Helmerson, Phillips, arXiv: 0805.3519



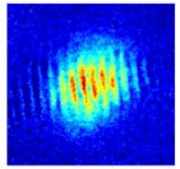
In situ images



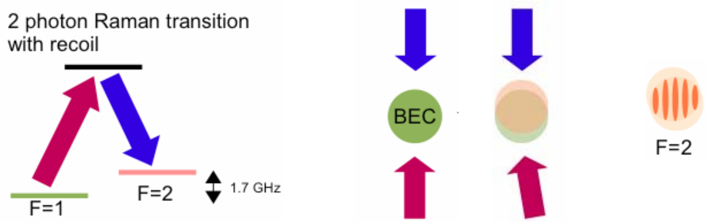
Number of atoms: 10^5 to $2\ 10^5$

Interaction coefficient: $\tilde{g} \sim 0.01$

Current studies at NIST



Study of quasi-long range coherence, by interfering the gas with a displaced copy of itself

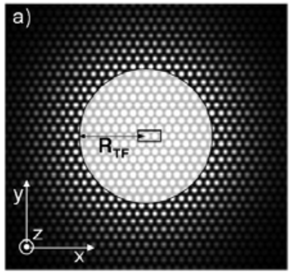


Point out the possible existence of three phases and not just two (see also Bisset-Blackie-Davis-Simula, arXiv:0804.0286)

- normal thermal phase
- intermediate quasi-condensate, no superfluidity (g_1 decays exponentially)
- superfluid quasi-condensate (below the KT transition point)

Another approach to the BKT mechanism in Boulder

V. Schweikhard, S. Tung, and E. A. Cornell, PRL **99,** 030401 (2007)



Array of parallel tubes at finite temperature T, coupled together by tunneling J

Equivalent to an array of Josephson junctions

Implementation of the discrete x-y model

Nice agreement with the BKT theory

Conclusions and prospects

- Evidence for a typical 2D behaviour: vortex proliferation
- Entangled role of condensation, superfluidity and spatial inhomogenity

  Finite size + harmonic trapping potential

  Experiments with 2D square boxes ?

- Experiments focused so far on density profiles and measurement of coherence properties

  Test of superfluidity ? Scissors mode, moment of inertia

- The dynamics of 2D gases has not been investigated yet experimentally

$$\langle \int e^{i\theta(\mathbf{r},t)}\, d^2r \rangle \;\propto\; (t/t_0)^{-T/8T_c}$$

Burkov et al., 2007