Summary/Revision Guide for the Financial Physics short option, TT2013

The lectures followed the plan set out in the **Intro + Contents**, which contains useful information that is not repeated here. Some of the lecture notes are the typescript of the book *Financial Market Complexity, what physics can tell us about market behaviour* by Johnson *et al.* Neil Johnson gave these lectures 2002-2006. **Relevant parts of the book by Johnson et al. are: Ch. 1, 2, 3, and Sections 6.1 to 6.3.**

Exam questions on past papers

The compilation of short option exam questions on the course website are marked to indicate irrelevant questions, e.g., material not covered this year (and therefore not examinable) includes Ch. 4 of the book by Johnson et al. and calculating the value of options in non-Gaussian markets (later part of Ch. 6).

Ch. 1 – definition of some financial terms, including market impact and demand

The temporal order of events in the market and the impact of demand on the price (explained diagrammatically in the lecture, Fig. 1.4).

There is much interesting information on Wikipedia - see Financial Market etc.

Details (not examinable) of the operation of markets, order books etc at: http://www.londonstockexchange.com/traders-and-brokers/traders-brokers/home.htm

Ch. 2 – statistical ideas and Black-Scholes equation

This chapter contains a lot of background statistical ideas (i.i.d. variables, PDFs) used in the rest of the course; these should be familiar from statistical mechanics, e.g. Brownian motion (aka Wiener process), random walks etc. Stochastic differential equations are introduced BUT there is no need to know how to solve them because in the derivation of the Black-Scholes equation (using Ito's lemma) the random variable is eliminated. Thus the value of the option is given by the solution of a partial differential equation. This equation can be put into the form of the diffusion equation describing the spreading of particle density or temperature (heat flow), i.e. the first derivative w.r.t. time depends on the second derivative w.r.t. x, the variable representing position in physics or price of the asset in finance). You need to have an overview of the method of solving the equation (various changes of variable and boundary conditions) but do not need the full mathematical details are too complicated. You should be able to sketch the values of call and put options at various times up to the expiry date for a nonzero interest rate and volatility. [Section 2.3 on Value-at-Risk is only relevant insofar as it underlines the importance of risk in finance.]

Ch. 3 - going beyond the standard theory (Econophysics)

Statistical analysis of market data by Stanley & Mantegna (and Mandelbrot) showed heavy-tailed distributions of prices changes, and other stylized facts indicating that financial markets have a more complex behaviour than a geometric random walk assumed in Ch. 2 (i.e. uncorrelated steps or price changes). Examples that illustrate the limitations of standard financial theory are a) the Parrondo effect, and b) a model with higher order temporal correlation of prices over 3 time steps but which gives the same mean and correlation over 2 time step as a random process. [N.B. only a bit of the general formalism in **3.5** is used later.]

Skip Ch. 4 and 5 of the book.

Ch. 6 – going beyond Black-Scholes. Pricing derivatives in the real world

Sections 6.1, 6.2 and 6.3 (Fig. 6.1 to 6.4) give an overview of how hedging works in practice. An 'implied-volatility smile' is shown in Fig. 6.11. Note also (for information only, not examinable) the analytic expression for implied volatility in eqn (6.84). This chapter serves to show the complexity of financial maths (quantitative finance).

LECTURE NOTES

Pricing of Forward Contracts, put-call parity

Introduction to effect of the interest rate on the *present value* an asset (discounting of prices, or a changing 'frame of reference' for prices). Put-call parity is derived by a no-arbitrage argument for a certain portfolio of options and assets.

The binomial tree model: a simple example of pricing financial derivatives (written in the form of an answer to a question).

In the simple coin-toss market (Fig. 2.2 in Johnson et al) there is equal probability of up and down movements hence no change in mean price (consistent with an interest rate of $r = 0$ for there to be no arbitrage). In the binomial tree model price changes are expressed as percentages (so the price cannot go negative) and the risk-neutral probability depends on the underlying interest rate. The lecture covered: risk-neutral probability for given interest rate, value of an option as a function of time, and hedging strategy.

Implied volatility can be deduced from option prices (VIX = CBOE Volatility Index where CBOE = Chicago Board Options Exchange).

Credit Default Swaps (non-examinable)

The pricing of these financial instruments based on the present value of expected payoff (another example of a no-arbitrage argument). A simple estimate was given followed by a more detailed treatment taken from the book by Hull (non-examinable). The risk-neutral probability of a credit event, or default, can be interfered from the pricing (market valuation); cf. deducing *implied volatility* from option prices.

Brownian motion (non-examinable). Treatment of Brownian motion using the Langevin equation, e.g. see third-year paper B1:I, Flows Fluctuations and Complexity (Prof. Marshall's lecture 15).

Recommended textbooks - see Introduction and Contents.

Prof. C. J. Foot, Oxford Physics, May 2013.