Credit Default Swaps

These notes borrow heavily from the treatment in Chapter 24 of the book by JC Hull entitled Options, Futures, and other Derivatives (8th ed. Pearson, sections 24.1 and 24.2).

Introduction

Credit Default Swaps are a particular kind of credit derivatives, and have been in the news in connection with the debt crisis in Greece (and the recent restructuring or default). Traditionally banks made loans to businesses (and individuals), using the money deposited by savers, and took the risk that the borrower would default (which was reasonable when averaged across many such transactions in a normal economic climate). Starting from the late 1990s, banks made extensive use of Credit Default Swaps (CDS) to shift the risk in their loans to other parts of the financial system and nowadays the contractual relationships between counterparties have become much more complex, i.e., a network of swaps between the different banks and between the banks and other counterparties such as insurance companies. Thus the financial institution bearing the credit risk is often not the institution that carried out the original credit check—this disjunction is not a good thing as became apparent in the credit crisis of 2007.

Definition of Credit Default Swaps

This is contract that provides insurance against the risk of a default by a particular company which is called the *reference entity.* Default by the reference entity is known as a *credit event.* The buyer of the CDS obtains the right to sell bonds issued by the company for their face value if a credit event occurs and the seller of the CDS agrees to buy those bonds for the principal amount that the issuer would repay at maturity (if it does not default). The total face value of the bonds that can be sold is known as the *notional principal* of the CDS. Note, however, that a CDS differs from an insurance policy in various ways including not having to own any of the underlying bonds in order to buy a CDS—so-called naked CDS are essentially a bet that the reference entity will default. Several European politicians accused naked CDS buyers of exacerbating the Greek financial crisis (see the Wikipedia article on CDS for more discussion and comments, e.g., by the financier George Soros). Countries can be the reference entities for CDS contracts which are called sovereign credit default swaps. Also unlike traditional insurance, swaps are much less regulated.

The buyer of the CDS makes regular payments to the seller until the end of the life of the CDS, or until a credit event occurs. We shall assume that these payments are made annually in arrears but it is straightforward to adapt the calculation for quarterly payments (which are more usual). As an example of a typical deal, suppose that two parties enter into a 5-year CDS on June 20, 2012 (which is one of the four standard maturity dates *viz.* March 20, June 20, Sept 20, Dec 20). Assume that the notional principal is \$10 m and the buyer agrees to pay 50 basis points per annum (0.5%), which is known as the CDS spread, i.e. \$500 000 at the end of years $1, 2, 3, 4$ and 5. (N.B. This is NOT equivalent to a total of \$2.5 m when interest is taken into account.) Payment stops when a credit event occurs but since payments is made in arrears there is usually a final accrual payment, e.g., if default occurs halfway through the year then half the annual payment is due (and we shall assume this case in the calculation below). A key aspect of a CDS contract is the definition of the credit event. This is usually defined as bankruptcy, restructuring of debt, or when the reference entity fails to make a payment when it becomes due. In March 2012 the International Swaps & Derivatives Association (ISDA), the body in charge of CDS, declared that Greece's 206 billion Euro (*£*171.5 bn) bond restructuring was a credit event. (Greece was given the first tranche of the financial bailout just a day before it was due to repay debts that matured on the March 20 deadline.) Although very topical (and there are many news articles online) the situation with respect to countries such as Greece is not a clear cut as for a company. When a credit event occurs the ISDA organises an auction process for the bonds that determines their value immediately after default. This value can be expressed as a fraction R of the face value of the bond; R is the recovery rate and therefore on notional principal of L the loss is $(1 - R)L$ and this is the payoff from the CDS, i.e., the amount that the seller of the CDS must pay to the buyer either in cash or other bonds depending on the contract. When the Lehman Brothers bank defaulted in Sept 2008, the recovery rate was only 8.625%, i.e., $1 - R = 0.91375$, and the total value of CDS contracts was \$400 billion, which was more than double the amount of outstanding debt of the bank.

Valuation of CDS with respect to bond yields and interest rates

Consider the following situation where a CDS is used to hedge a position in a corporate bond. An investor buys a 5-year bond yielding 7% per year and at the same time a 5-year CDS as protection against default of the issuer of the bond. If the CDS spread is 200 basis points, or 2% per annum, then the investor earns 5%

nett. This is risk-free and so must be approximately equal to risk-free interest rate available in the market (which is taken to be LIBOR, the London Interbank Offered Rate; see Wikipedia for further info). If this were not true then there would be an arbitrage opportunity, i.e., borrowing money to buy the bond and CDS, or short selling, depending on the sign of the difference.

Valuation of CDS

The CDS can be calculated from the estimated probability of default of the reference entity. Note that this is the risk-neutral default probability *not* the real-world probability—the difference between these was explained for the binomial tree model (which also uses risk-neutral probabilities) and is discussed in Chapter 23 of Hull's book (ibid.). [Real-world, or physical, probabilities are lower than the risk-neutral values, e.g., a historical hazard rate of 0.04% per annum for Aaa-rated stock; other examples are given in Table 23.4 in Hull's book.] Table 1 shows the survival probabilities for the case where the risk-neutral probability of default during a year conditional on no earlier default is 2%. The probability of default in the first year is $p = 0.02$ and the probability of survival until the end of the first year is $1 - p = 0.98$. The probability of default during the second year is $p(1-p) = 0.02 \times 0.98 = 0.0196$ and probability of survival until the end of the year is $(1-p)^2 = 0.9604$. And so on to fill in the rest of Table 1.

Table 1: Unconditional default and survival probabilities

Time (years)	Default probability	Survival probability
1	0.0200	0.9800
2	0.0196	0.9604
3	0.0192	0.9412
4	0.0188	0.9224
5	0.0184	0.9039

The further parameters we shall assume to calculate the present value of the CDS are:

i) the risk-free interest rate (LIBOR) is 5% p.a. with continuous compounding;

ii) a recovery rate of $R = 40\%$, after a credit event;

iii) defaults happen halfway through a year; and

iv) CDS payments are made annually, at the end of the year.

All the numbers are given for a notional principal of \$1.

A simple estimate can be made by setting the annual payment s equal to the expected payoff (p.a.) given by the default rate λ multiplied by $1 - R$:

$$s = \lambda \times (1 - R) = 0.02 \times 0.6 = 0.012. \tag{1}$$

The approximations used in this estimate are apparent when it is compared with the following more complete calculation.

The three parts of the calculation are shown in Tables 2, 3, and 4 respectively.

From Tables 2 and 4, the present value of the expected payments is $4.0704s + 0.0426s = 4.1130s$. Notice that the accrual payment is only a small contribution (about 1% of the total) and so assumption iii has little influence. Equating this to the total expected payoff from Table 3, we find

$$\begin{aligned} 4.1130s &= 0.0511 \\ s &= 0.0124 \end{aligned}$$

Thus the CDS spread is 124 basis points per year, e.g., for a notional principal of \$10 m the payments are \$124 000 per year. This is close the rough estimate in eqn (1) of 120 bp per annum (but calculations in accounting need to be done with high precision).

Table 2: Calculation of the present value (PV) of expected payments that are made at rate of s per year. For example, the probability of the third payment being made is 0.9412; therefore the expected payment is $0.9412s$ and its present value is $0.9412s\, e^{-0.05\times 3} = 0.8101s$. The total present value of the expected payments over five years is $4.0704s$, cf. $5s$ in the limit where the interest rate and default rate both tend to zero.

Time (years)	Probability of survival	Expected payment	Discount factor	PV of expected payment
1	0.9800	$0.9800s$	0.9512	$0.9322s$
2	0.9604	$0.9604s$	0.9048	$0.8690s$
3	0.9412	$0.9412s$	0.8607	$0.8101s$
4	0.9224	0.9224v	0.8187	$0.7552s$
5	0.9039	$0.9039s$	0.7788	$0.7040s$
Total				$4.0704s$

Table 3: Calculation of the present value of the expected payoff. E.g., the probability of a payoff in the middle of the third year is 0.0192; thus for a recovery rate of 40% (of the notional principal of \$1) the expected payoff is $0.0192 \times 0.6 \times 1 = 0.0115$; the present value of this payoff is $0.115e^{-0.05\times 2.5} = 0.0102$.

Time (years)	Probability of default	Recovery rate	Expected payoff (\$)	Discount factor	PV of expected payoff (\$)
0.5	0.0200	0.4	0.0120	0.9753	0.0117
1.5	0.0196	0.4	0.0118	0.9277	0.0109
2.5	0.0192	0.4	0.0115	0.8825	0.0102
3.5	0.0188	0.4	0.0113	0.8395	0.0095
4.5	0.0184	0.4	0.0111	0.7985	0.0088
Total					0.0511

Table 4: Calculation of the present value of the accrual payments made in the event of a default. The probability of default in (the middle of) the third year is 0.0192; the accrual payment for the half year is $0.5s$; the expected payment is $0.0192 \times 0.5s$ which has present value of $0.0096s\, e^{-0.05\times 2.5} = 0.0085s$. Similar calculations for the other years give a total expected accrual payment of $0.0426s$.

Time (years)	Probability of default	Expected payoff (\$)	Discount factor	PV of expected payoff (\$)
0.5	0.0200	$0.0100s$	0.9753	$0.0097s$
1.5	0.0196	$0.0098s$	0.9277	$0.0091s$
2.5	0.0192	$0.0096s$	0.8825	$0.0085s$
3.5	0.0188	$0.0094s$	0.8395	$0.0079s$
4.5	0.0184	$0.0092s$	0.7985	$0.0074s$
Total				$0.0426s$

Estimating Default Probabilities

It is worth reiterating that the default probabilities used in the Table 1 and the subsequent tables based upon it, are risk-neutral not real-world probabilities. The risk-neutral probabilities can be estimated from CDS quotes by reverse-engineering the above calculations. This is in the same spirit as implying volatility by working backwards from the price of options. (The example given in Hull's book is that of a 5-year CDS with a CDS spread of 100 basis points which implies a default probability, conditional on no earlier default, of 1.61% p.a.).

An important point about the CDS market is that some market participants are likely to have more information about the creditworthiness (hence default probability) than others, e.g., banks working with a particular company (or country). This is called the asymmetric information problem, and so one has to be cautious about applying usual market assumptions.

As already mentioned, these notes follow closely the treatment in the book by JC Hull entitled *Options, Futures, and other Derivatives* (8th ed. Pearson) Chapter 24; the rest of these notes summarise Chapter 23 on **Credit Risk**. Credit risk arises from the possibility that borrowers and counterparties in derivatives transactions may default.

Credit Ratings

There are a number of rating agencies such as Moodys, S&P, and Fitch. They use slightly differ notation, e.g. starting from the highest Moodys use Aaa, Aa, A, Baa for investment grade bonds, and whereas the others use AAA, AA, A, BBB. The rating agencies produce data on the default rates, e.g., the cumulative default rate after 10 years is 0.497% for AAA and 4.851% for Baa. The probability of default $Q(t)$ is given by $Q(t) = 1 - e^{-\lambda^* t}$ Where λ^* is the real-world average hazard rate (default intensity) over the period (cf. radioactive decay), which is less than the risk-neutral value λ used in eqn (1). The historical, or physical, default probabilities are much lower that the risk-neutral values.

Quanto CDS

Financial engineers have a penchant for creating ever more complicated instruments. The term quanto is used to denote derivatives where the payoff is associated with variables in one currency but it paid in another currency, e.g., a CDS on German (or UK) government bonds with a payoff in dollars. For any country in the Euro zone, a Euro-denominated sovereign CDS will be cheaper than the dollar-denominated sovereign CDS, e.g., CDS on German government bonds with a payoff in Euros will be cheaper than one where the payoff is in dollars because if Germany were to default then the Euro would lose value. Interestingly, we can deduce from the difference in the CDS spread of these two types of sovereign CDS what the market expects to happen in the event of a default. The difference in the spreads for Euro- and dollar-denominated sovereign CDS (in Nov. 2010) was 36% for German bonds and 7% for Greek bonds, thus indicating the traders assessment that default by Germany would have a much greater impact on the Euro-dollar exchange rate than a default by Greece. (Other countries in the Euro zone lie in the range 20 - 35%.) However one should be careful about reading too much into such figures since usual market behaviour may not apply to countries—defaults are not independent and it is likely that a Euro crisis would affect many countries (contagion).

Implied sovereign default probabilities online can be found at `http://www.dbresearch.com`. Deutsche Bank. Menu=Research; choose Emerging Markets/Country Risk. Online Tools (right of screen); choose CDS tool.

Credit Value Adjustment (CVA)

So far we have dealt with credit risk on loans, or bonds, but there is also a credit exposure in derivatives transactions, i.e., if we buy an option from an institution that goes bankrupt before we excise the option. This more difficult to price than CDS because of the various possible outcomes, e.g., if the market conditions are such that we would not exercise the option anyway then we do not lose (any more than the initial value). The credit value adjustment is the correction term, CVA, that must be applied to account for the counterparty defaulting. This is a negative adjustment since we lose money. However this unilateral CVA assumes that we do not have any credit risk, but in reality both counterparties may default. Therefore it is common to compute a bilateral CVA which contains a positive contribution, Debit Valuation Adjustment (DVA), as well as the negative CVA term. Bilateral Valuation Adjustment, BVA = DVA-CVA. This leads to the interesting consequence that a bank can benefit from a deterioration in its own credit quality, i.e., when value at risk is reduced on its balance sheet because of the possibility of the bank itself defaulting!