

Summary: An introduction to the physics-based approach to finance theory (so-called ‘Econophysics’). This interdisciplinary field aims to apply ideas and mathematical techniques developed in physics (particularly those related to statistical mechanics) to improve our understanding of the empirically observed fluctuations in global financial markets. Emphasis is placed on the extent to which asset prices deviate from random walk behaviour, the development of microscopic models which may help to explain these deviations, the minimization or *hedging* of financial risk and the phenomenon of financial crashes. The course covers the main shortcomings of standard finance theory, new approaches and techniques introduced by physicists and some ideas concerning complex systems.

Introduction

This course follows the book written by the previous lecturer Neil Johnson (and his co-authors) [1]; the lecture notes are essentially the typescript of the book (*excluding* chapter 5 of the book; lecture handout 5 contains new examinable material). Chapters 6 and 7 of the book contain more advanced material with some very useful insights. The exam questions on the Short Option papers: 2002 -2006 (inclusive) are useful for revision and this year’s paper will be similar in style (and content). The ordering of material in lectures will be (where numbers refer to sections of the book): this Introduction; 1 to 2.4.2; 6.1, 6.2 (Figs. 6.1 – 6.4); The binomial tree model; 2.4.3 to 4.8.2, part of chapters 6 and 7 will be covered). The only material not directly contained in Johnson et al’s book is: i) this *Introduction*, and ii) *The binomial tree model: a simple example of pricing financial derivatives* which closely follows reference [2].

This course uses the same notation as in the book and past exams (with a few exceptions) but this appears not to be standard notation in the field, e.g. as found on the Wikipedia entry about the Black-Scholes equation, or the course on Financial Mathematics given in the Maths Dept [2]. The course in Maths is more conventional (and more restricted in scope)—it involves a rigorous treatment of the Black-Scholes equation and a lot of the fiddly mathematical details that arise when applying it various cases—whereas this course concentrates on understanding the principles in the simplest case. References [3-6] are for interest only (NOT part of this course); these papers by Neil Johnson and coworkers illustrate the great range of applicability of the ideas of complexity. There are many other fascinating papers in econophysics, e.g. by Dooyne Farmer at the Santa Fe Institute (arXiv:1005.4976, *An empirical study of the tails of mutual fund size*, and arXiv:0908.1555, *Leverage Causes Fat Tails and Clustered Volatility*). Finally, it is worth noting that there are strong critics of the (mis)application of financial mathematics, e.g. by Nassim Taleb author of *The Black Swan*, and there is a vigorous ongoing debate between econophysicists and economists.

[1] Financial Market Complexity: What Physics can tell us about market behaviour. Oxford University Press, 2003 ISBN: 0198526652 by Neil F. Johnson, P. Jefferies and Pak Ming Hui.

[2] B10b Mathematical Models of Financial Derivatives. <http://www.maths.ox.ac.uk/courses/course/12458/material> Especially useful for this course are the two summaries: Lecture note 6 and Lecture note 8. (The slides used in those lectures can also be downloaded.)

[3] Financial systems: Ecology and Economics. Neil Johnson and Thomas Lux. *Nature* **469**, 302 (2011)

[4] Predictability of large future changes in a competitive evolving population. D.Lamper, S.Howison, N.F. Johnson. *Phys. Rev. Lett.* **88**, 017902 (2002), arXiv:cond-mat/0105258, or <http://www.economist.com/node/638530>.

[5] Dynamic Red Queen explains patterns in fatal insurgent attacks. Neil Johnson et al. arXiv:1101.0987

[6] How does Europe Make Its Mind Up? Connections, cliques, and compatibility between countries in the Eurovision Song Contest. D. Fenn, O. Suleman, J. Efstathiou and N. F. Johnson. arXiv:physics/0505071

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Lecture Notes 5 (NOT from Johnson et al [2])

The binomial tree model: a simple example of pricing financial derivatives

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- 7.4 Predicting the future: Who wants to be a Millionaire?

Texts referred to in the handouts (These are not required reading; just listed for interest)

- [WDH] *The mathematics of financial derivatives*, P. Wilmott, J.Dewynne and S.Howison (CUP, 1996).
- [BP] *Theory of financial risks*, J.P. Bouchaud and M. Potters (CUP, 2000).
- [MS] *An introduction to Econophysics*, R.N. Mantegna and H.E. Stanley (CUP 2000).
- [W] *Derivatives*, P. Wilmott (Wiley, 1998).
- [G] *The nature of mathematical modelling*, N. Gershenfeld (CUP, 1999).
- [F] *An introduction to probability theory and its applications*, W. Feller (Wiley, 1968).
- [V] *The statistical mechanics of financial markets*, J. Voit (Springer, 2001).
- [S] *Non-linear dynamics and chaos*, S. Strogatz (Perseus, 2001).
- [So] *Critical phenomena in natural sciences: chaos, fractals, self-organization and disorder*, D. Sornette (Springer, 2000).
- [CLM] *The econometrics of financial markets*, J.Y. Campbell, A.W. Lo and A.C. MacKinlay (Princeton University Press, 1997).
- [P] *Finance & financial markets*, K. Pilbeam (Macmillan business, 1998).