8. A source produces polarization entangled photon pairs in the Bell state $|\Psi^+\rangle = |V\rangle_a|H\rangle_b + |H\rangle_a|V\rangle_b$. Photon a is sent to Alice and photon b to Bob who perform polarization measurements. Alice can measure at angles $\phi_a^1 = 0$, $\phi_a^2 = \pi/4$, or $\phi_a^3 = \pi/8$ to the vertical, while Bob can perform measurements at angles $\phi_b^1 = 0$, $\phi_b^2 = -\pi/8$, or $\phi_b^3 = \pi/8$.

The polarization states are identified with logical qubit states $|V\rangle \equiv |0\rangle$ and $|H\rangle \equiv |1\rangle$, and thus a measurement at an angle $\phi = 0$ corresponds to measuring the Pauli operator σ_z for the relevant photon. Calculate the operators that are measured by Alice when her detector is set at angles ϕ_a^i, and those that are measured by Bob when his detector is set at angles ϕ_b^j. [4]

Explain in detail a protocol for Alice and Bob to establish a secret key from their measurement results. Also discuss how this protocol allows them to calculate a quantity S whose value can exclude the possibility of an eavesdropper being present in an ideal setup. Give qualitative physical reasons why an eavesdropper cannot evade detection in this scheme. [15]

An eavesdropper adopts a simple intercept resend strategy. She measures photon b at an angle $\phi_e = 0$ and re-sends the photon in the measured state. How does S change in comparison to the ideal scheme? [6]