7. Explain **two** of the following quantum information protocols and outline the role and working of the device listed in brackets in a quantum optical implementation:

(a) Quantum dense coding (BBO crystal),

(b) Entanglement swapping (Bell state analyzer),

(c) Key distribution with EPR pairs (single photon detectors). [8]

Alice and Bob share an entangled pair of qubits in the state $|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ and Alice wants to use this EPR pair, a perfect Bell state analyzer and a classical communication channel to transmit an unknown state $|\psi\rangle$ of a third qubit to Bob. Bob is able to apply any single-qubit operation to his qubit. Describe and explain a protocol for achieving this, giving the three-qubit state after each step in the protocol. How much classical information needs to be transmitted over the classical channel to transmit one qubit? [10]

Finally assume that Alice has an imperfect Bell state analyzer which cannot distinguish the states $|\phi^+\rangle$ and $|\phi^-\rangle = (|00\rangle - |11\rangle)/\sqrt{2}$. She does not tell Bob about this imperfection but randomly assumes one of the two states whenever the Bell state analyzer gives an ambiguous result. Calculate the fidelity with which an arbitrary state $|\psi\rangle$ is teleported in this case. Which states are teleported with maximum fidelity and which states are teleported with minimum fidelity? [7]

8. Explain the properties of universal sets of quantum gates and their importance in quantum computing. Give an example of a set of quantum gates which is universal.

The Hamiltonian describing a collisional phase gate between two qubits (ignoring all single-particle contributions) is given by

$$H = \Omega|01\rangle\langle 01|,$$

where Ω is a measure of the interaction strength. How does each computational basis state evolve in time under H? Write down a matrix description of the resulting phase gate U_ϕ when H is turned on for a time τ, and determine how the phase ϕ depends on τ. Draw a quantum network which implements a controlled-NOT gate using only single qubit gates and U_π. [10]

Suppose the phase gate is applied to two qubits in state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, followed by a Hadamard gate applied to just the first qubit. Show that when $\phi = n\pi$ the resulting state is a product state for even n and a Bell state for odd n.

Three qubits are initially in state $|+\rangle$ and phase gates with phase $\phi = n\pi$ are applied between qubits 1 and 2 and between qubits 2 and 3 followed by a Hadamard gate on the first and third qubits. Show that the resulting state is a product state for even n and a GHZ state for odd n. [10]

A measurement of the first qubit of this GHZ state is made in the $\{|+\rangle, |-\rangle\}$ basis. What is the resulting state of the two remaining qubits if the outcome of this measurement is not revealed? Show that this state is not entangled. [5]