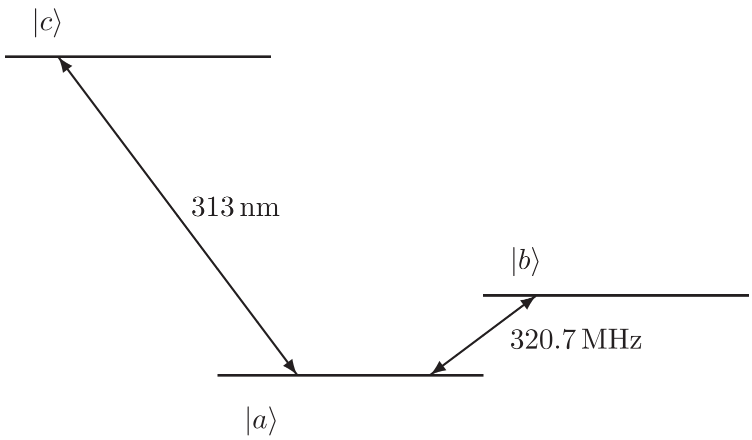
5. The diagram shows schematically three energy eigenstates in the ion $^9\mathrm{Be}^+$. States $|a\rangle$ and $|b\rangle$ are hyperfine states of the level 2s $^2S_{1/2}$, while $|c\rangle$ is a hyperfine state from 2p $^2P_{3/2}$. There are allowed transitions between the states as indicated.



Suppose an ion begins in $|a\rangle$, and that RF radiation is applied at a frequency of 320.7 MHz. Show, stating any approximations you make, that the ion undergoes coherent oscillations between $|a\rangle$ and $|b\rangle$. Of the states shown, why might these two be a good candidate for implementing a qubit? [10]

In an experiment on this ion the RF field was adjusted so that a NOT gate took 256 ms. Explain why relaxation may be neglected during this time. Estimate how closely the frequency of the RF radiation must be matched to the transition to get an accurate NOT gate. [5]

The system can also be excited from $|a\rangle$ to $|c\rangle$ using 313 nm laser light, resulting in strong fluorescence at 313 nm. If fluorescence is observed then the system must have been in $|a\rangle$; thus the application of 313 nm light acts as an abrupt measurement in the $\{|a\rangle, |b\rangle\}$ basis. Show that if the state of the ion is measured at n equally spaced times during the implementation of a NOT gate, then an ion starting in $|a\rangle$ will almost certainly remain there if n is large (the quantum Zeno effect) and estimate the probability of its leaving the state for the case $n = 64$. [10]