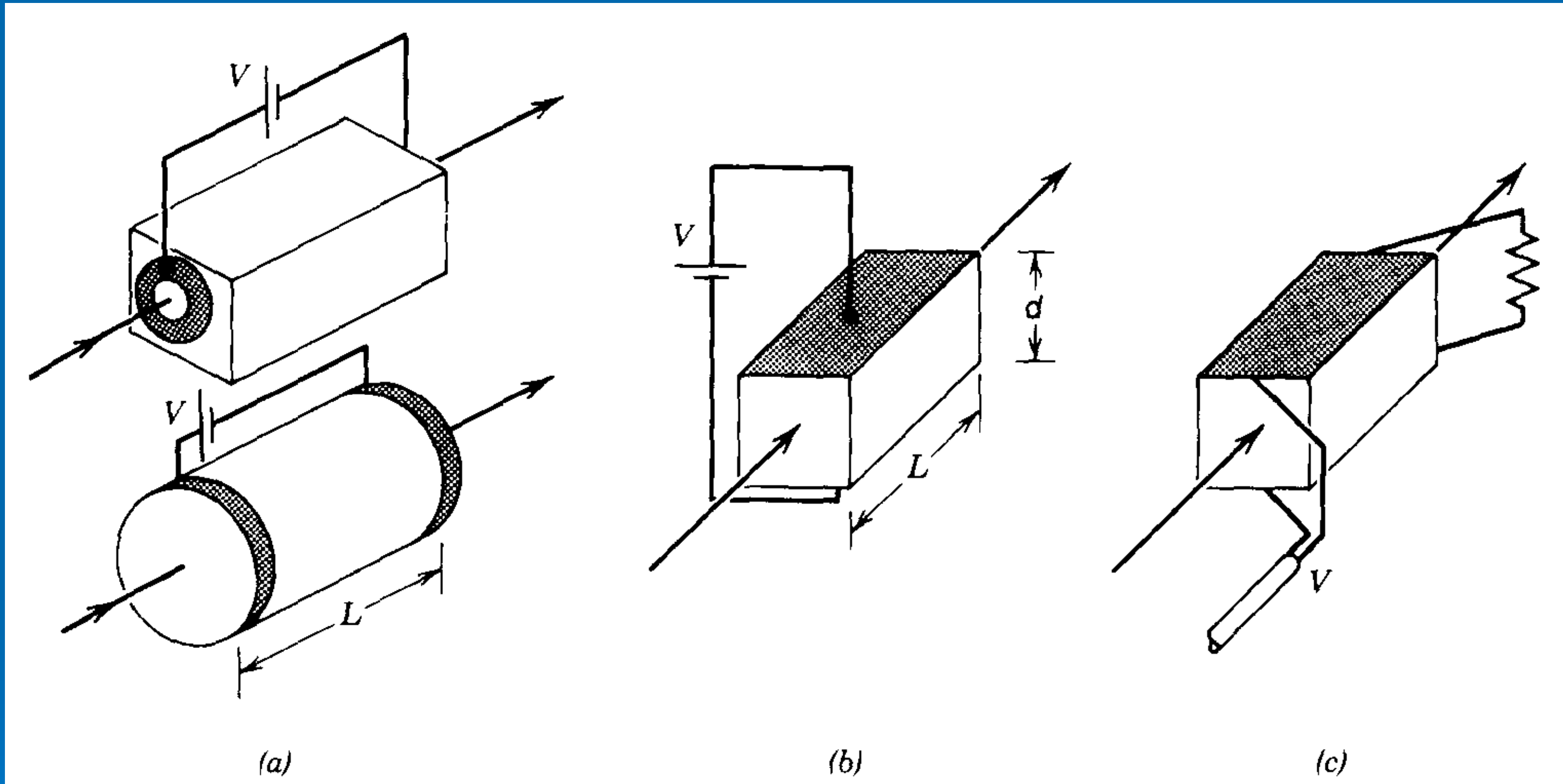
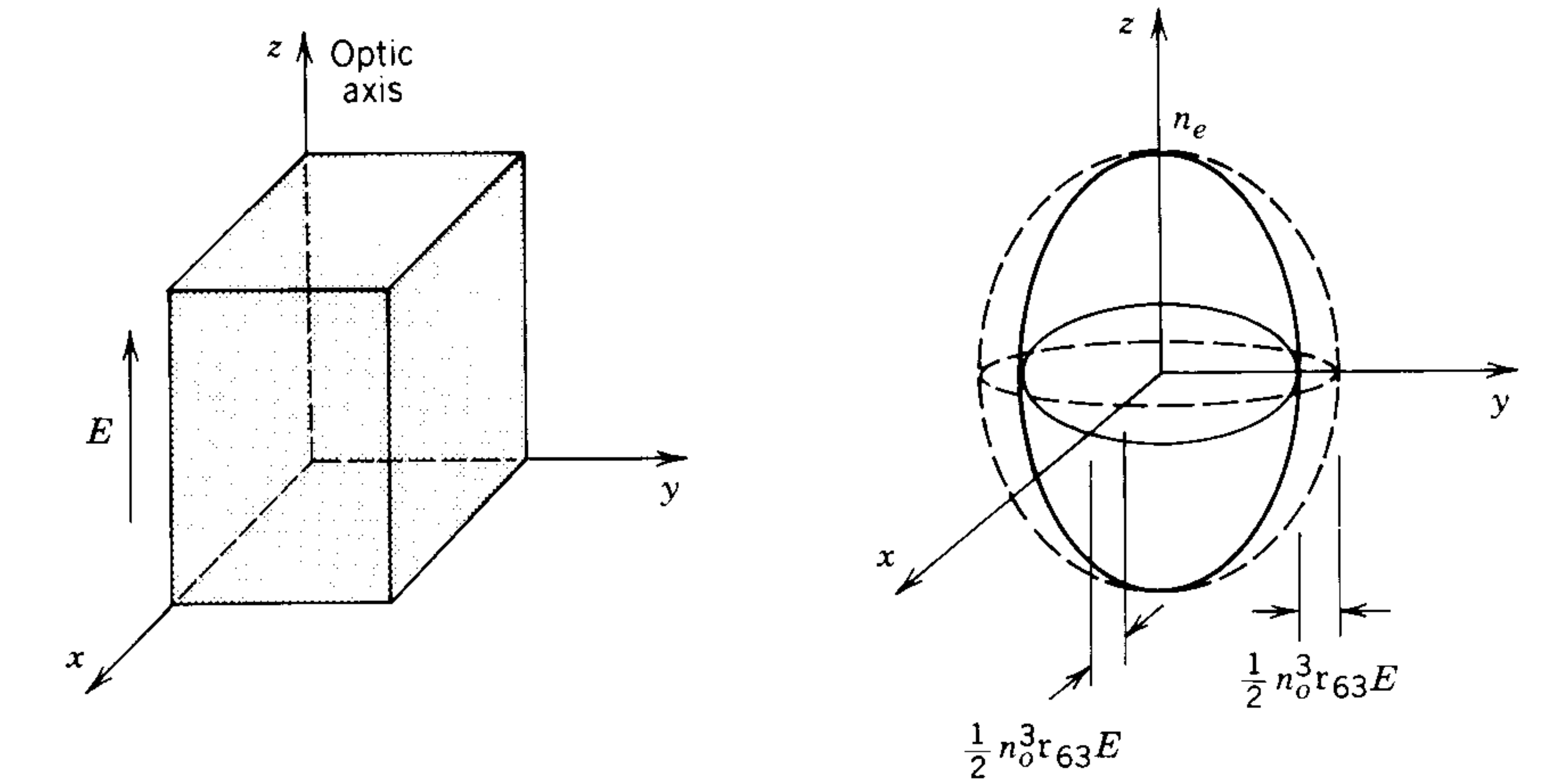
Modulators & SHG

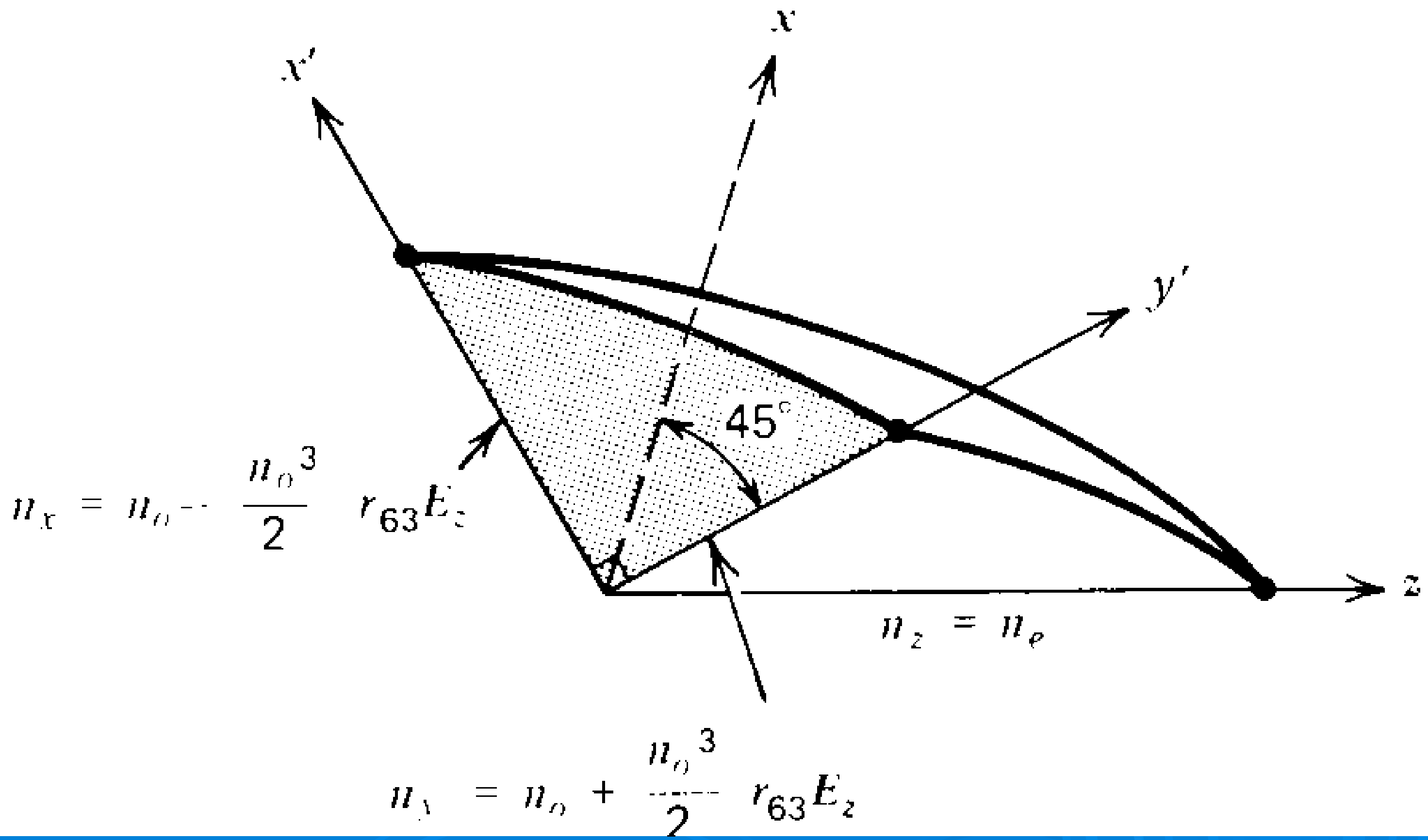


(a) Longitudinal field; (b) Transverse field; (c) Travelling-wave field.

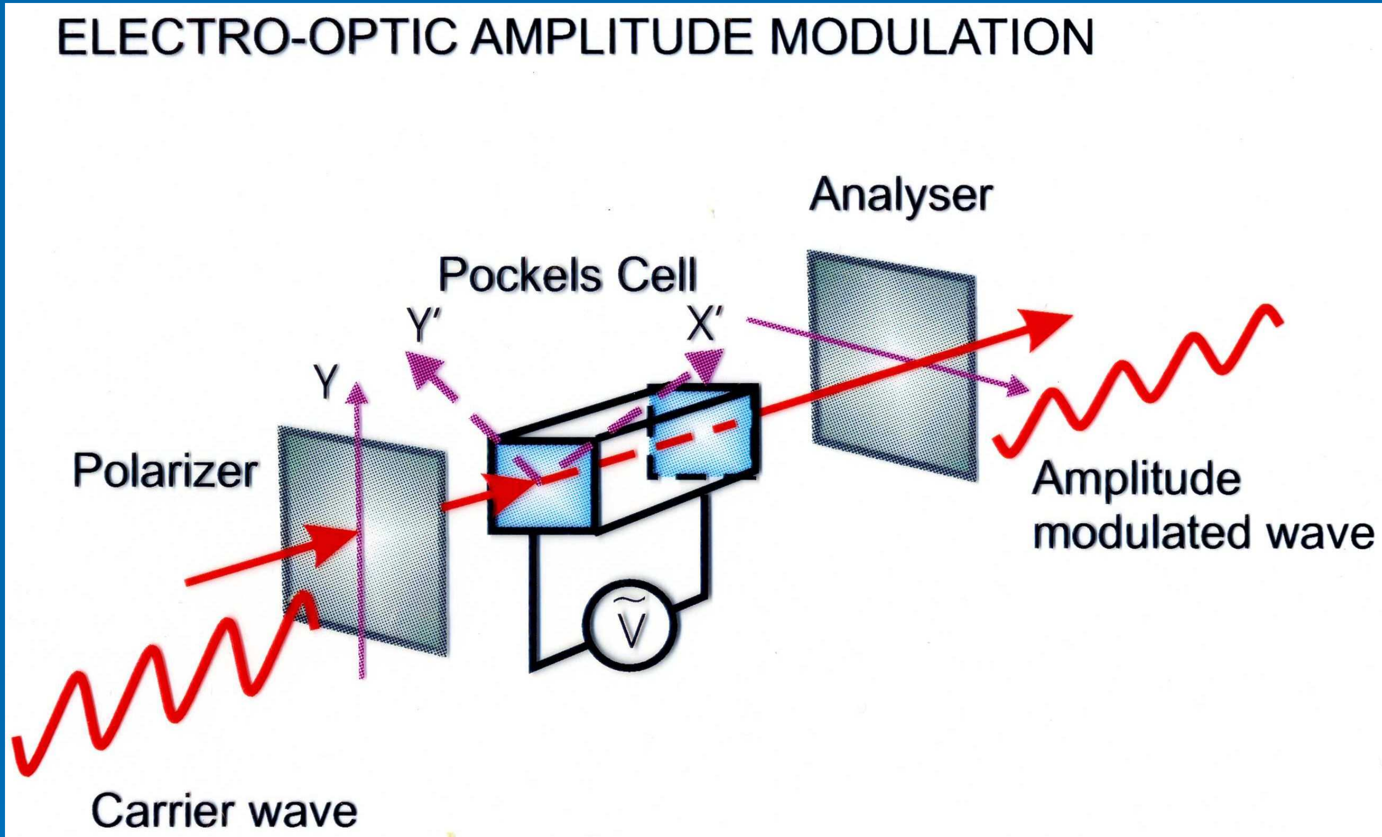
Pockels Effect in ADP



The index ellipsoid



The Longitudinal Modulator



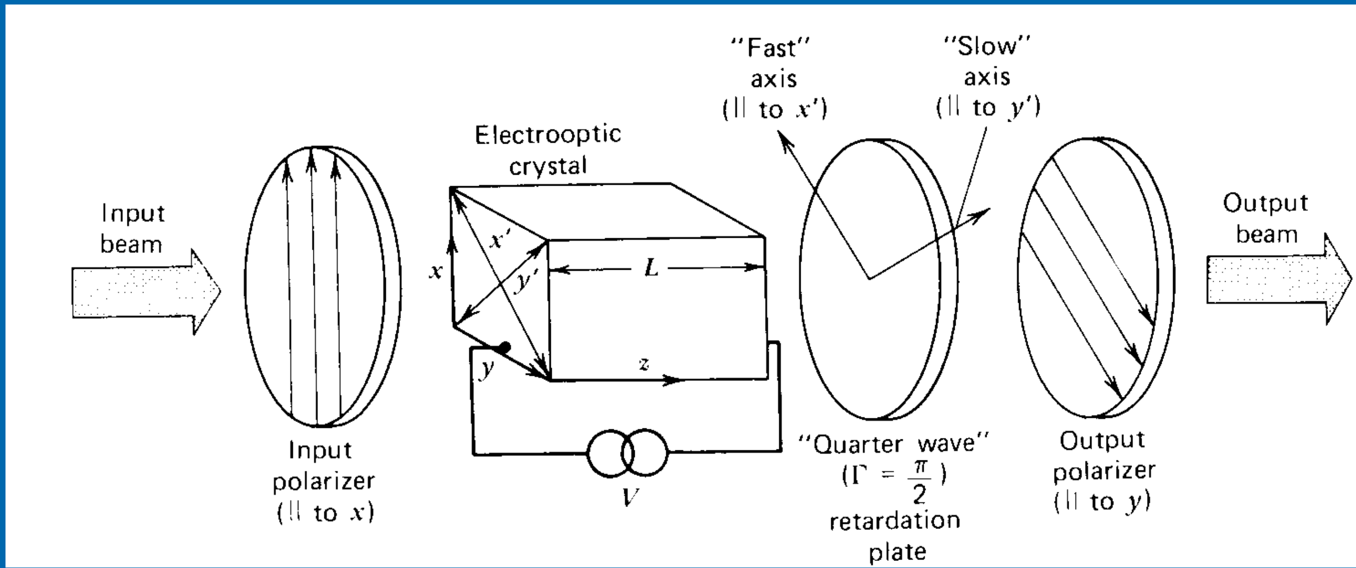
Output field

$$E_{x'}(\ell) = A; \qquad E_{y'}(\ell) = A\exp(-i\phi)$$
$$E_y(\ell) = \frac{A}{\sqrt{2}}\left[\exp(-i\phi) - 1\right]$$

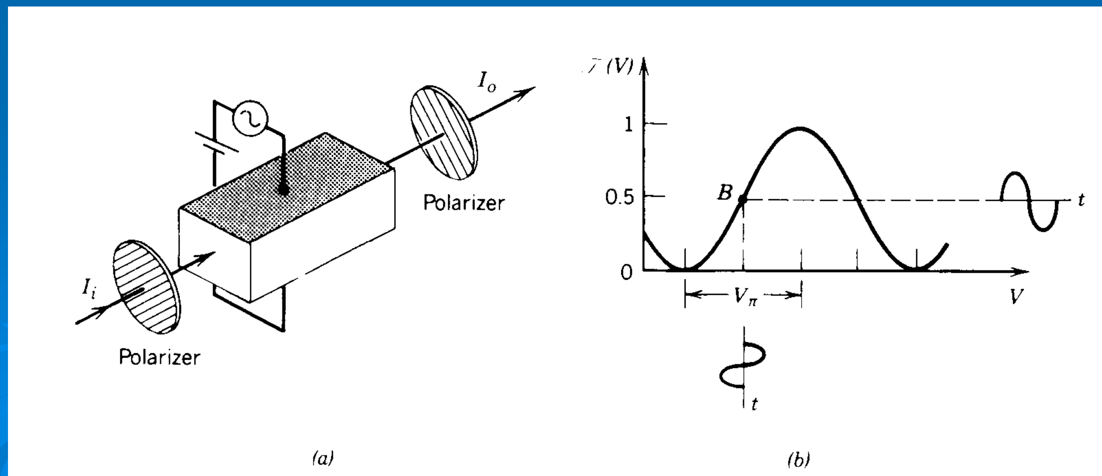
Input field

$$E_{x'} = E_{y'} = A; \quad E_x = 0$$

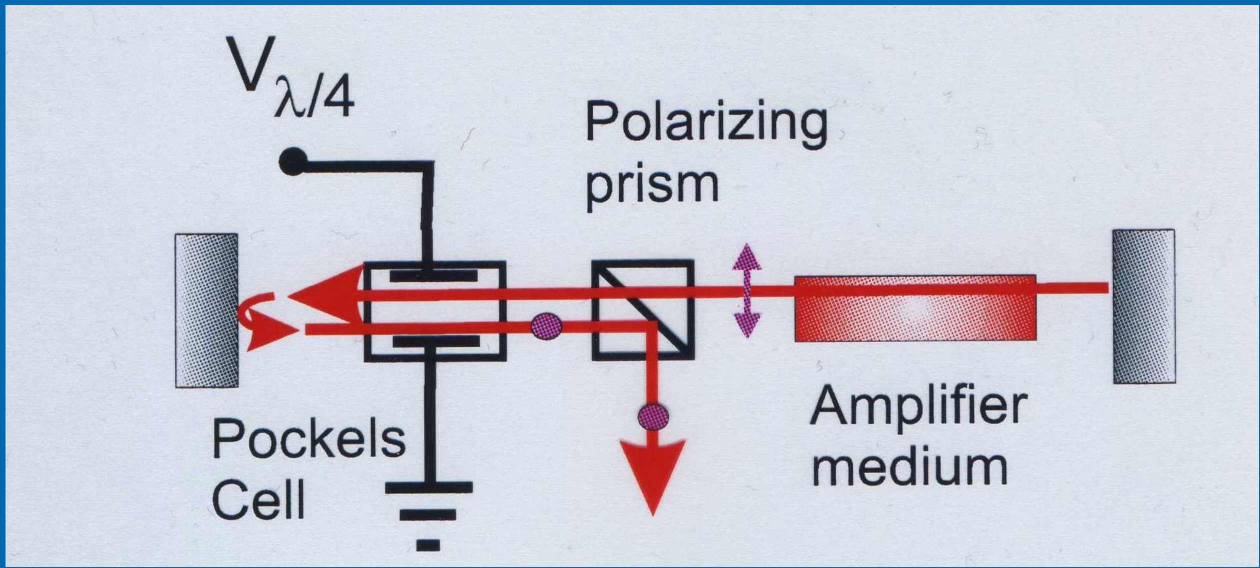
Amplitude modulation (Transverse)

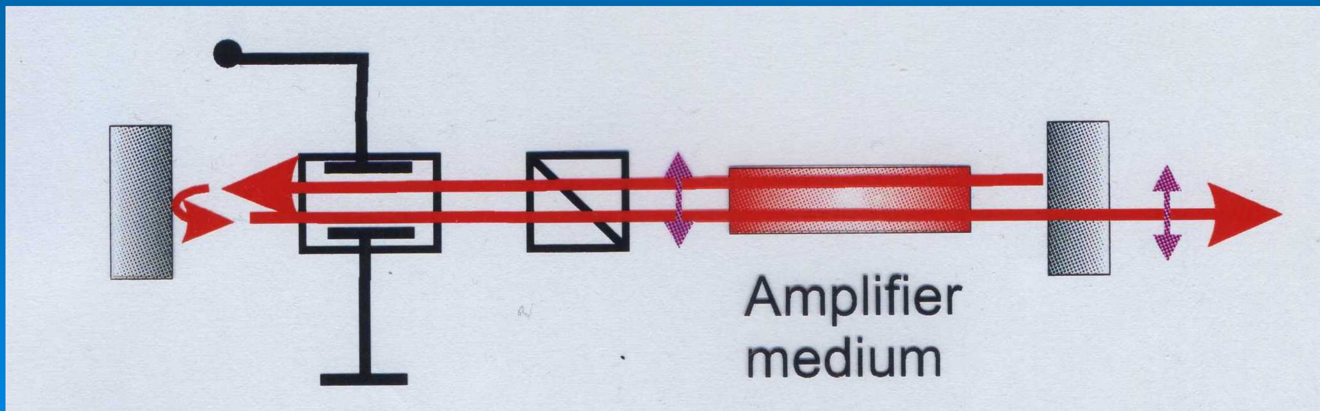


Inserting a quarter-wave plate provides a static bias to point *B* providing a near linear response

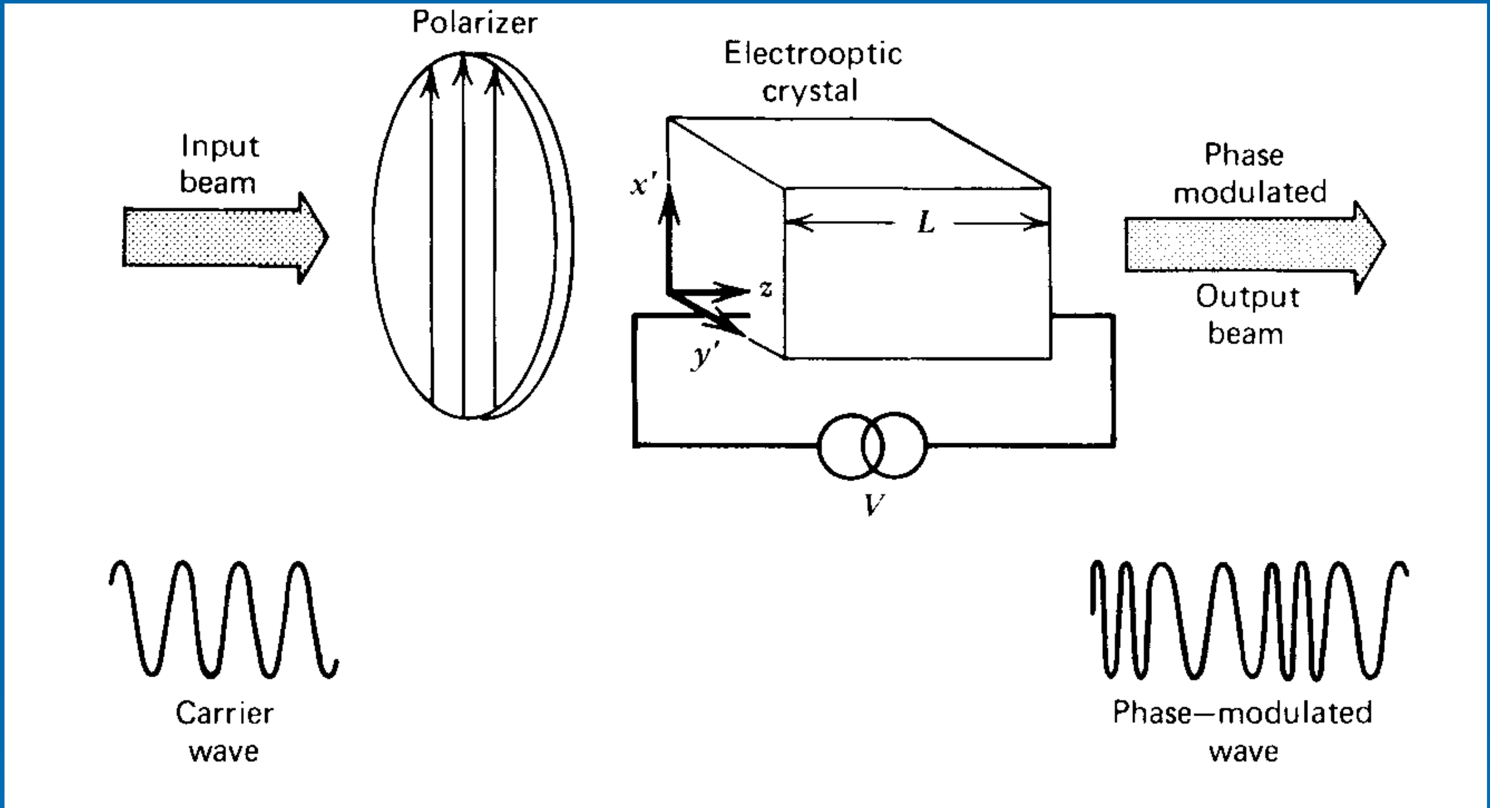


Application: Q-switching





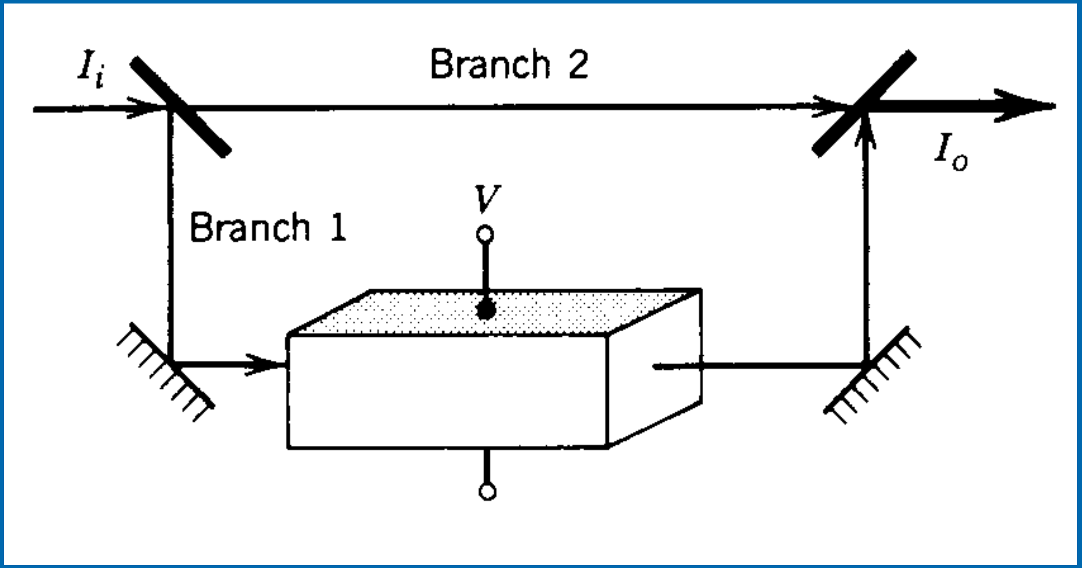
Phase modulation



$$E_{in} = A \cos \omega t$$

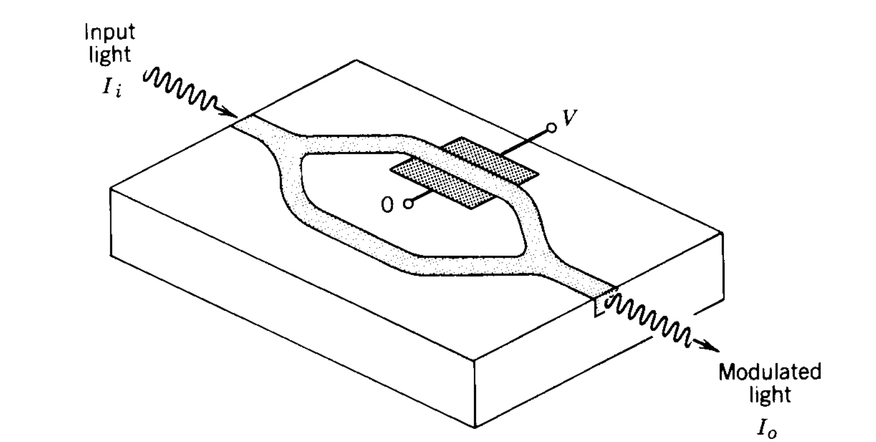
$$E_{out} = A \cos(\omega t - kx + \Delta\phi) = A \cos(\omega t - \frac{\omega}{c}(n_0 - \frac{n_0^3}{2} r_{63} E_m \sin \Omega t)\ell)$$

Phase modulation



Mach-Zehnder interferometer with a phase modulator in one arm

Optical fibre version of the same interferometer



Phase modulation: the maths 1

The input & output light fields

$$\begin{aligned} E_{in} &= A\cos\omega t \\ E_{out} &= A\cos(\omega t - kx + \Delta\phi) \end{aligned}$$

$$= A\cos(\omega t - \frac{\omega}{c}(n_0 - \frac{n_0^3}{2} r_{63} E_m \sin\Omega t)\ell)$$

And the voltage on the crystal is:

$$V_m = V_0 \sin\Omega t$$

$$\begin{aligned} \Delta\phi &= \frac{\omega\ell}{c}\Delta n = \frac{\omega\ell}{c} \times \frac{1}{2} n_0^3 r_{63} \times \frac{V}{\ell} \\ \Longrightarrow & \quad \delta = \frac{1}{2c} n_0^3 r_{63} \omega_0 V_0 \end{aligned}$$

Phase modulation: the maths 2

Expanding the cosine:

$$A\cos(A+B) = A\cos\omega_0 t\cos(\delta\sin\Omega t) - A\sin\omega_0 t\sin(\delta\sin\Omega t)$$

And using the Bessel function identity:

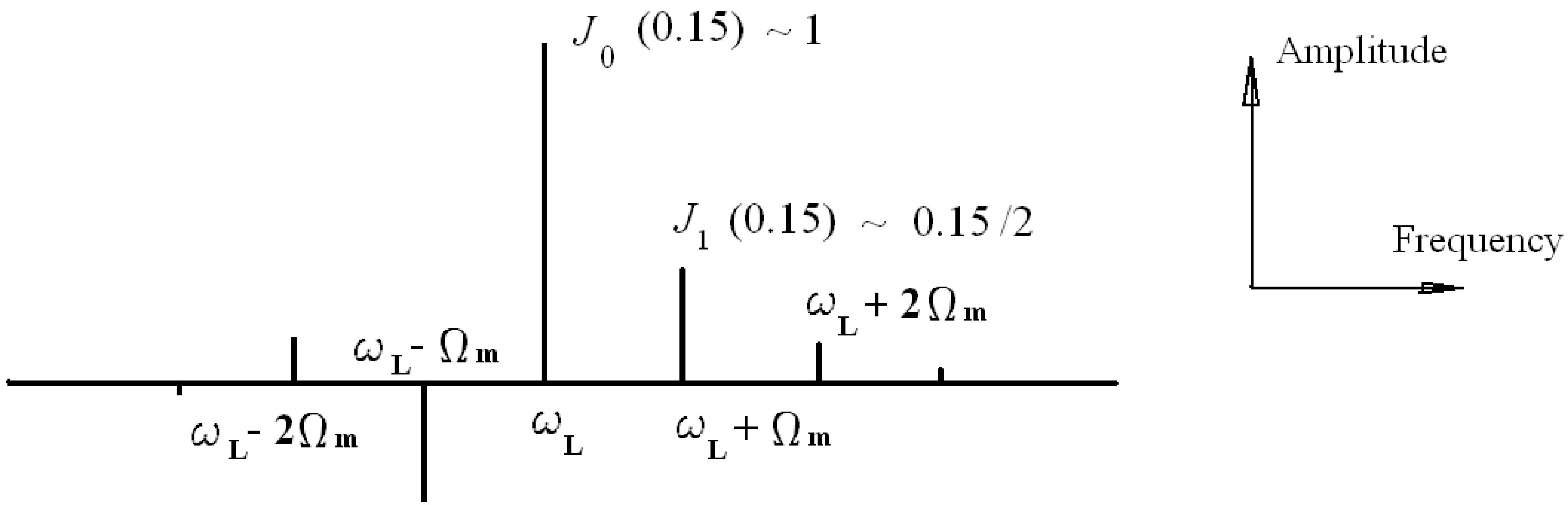
$$\exp\{ix\sin\theta\} = J_0(x) + 2iJ_1(x)\sin\theta + 2J_2(x)\cos\theta....$$

Gives for the output field – a series of side-bands

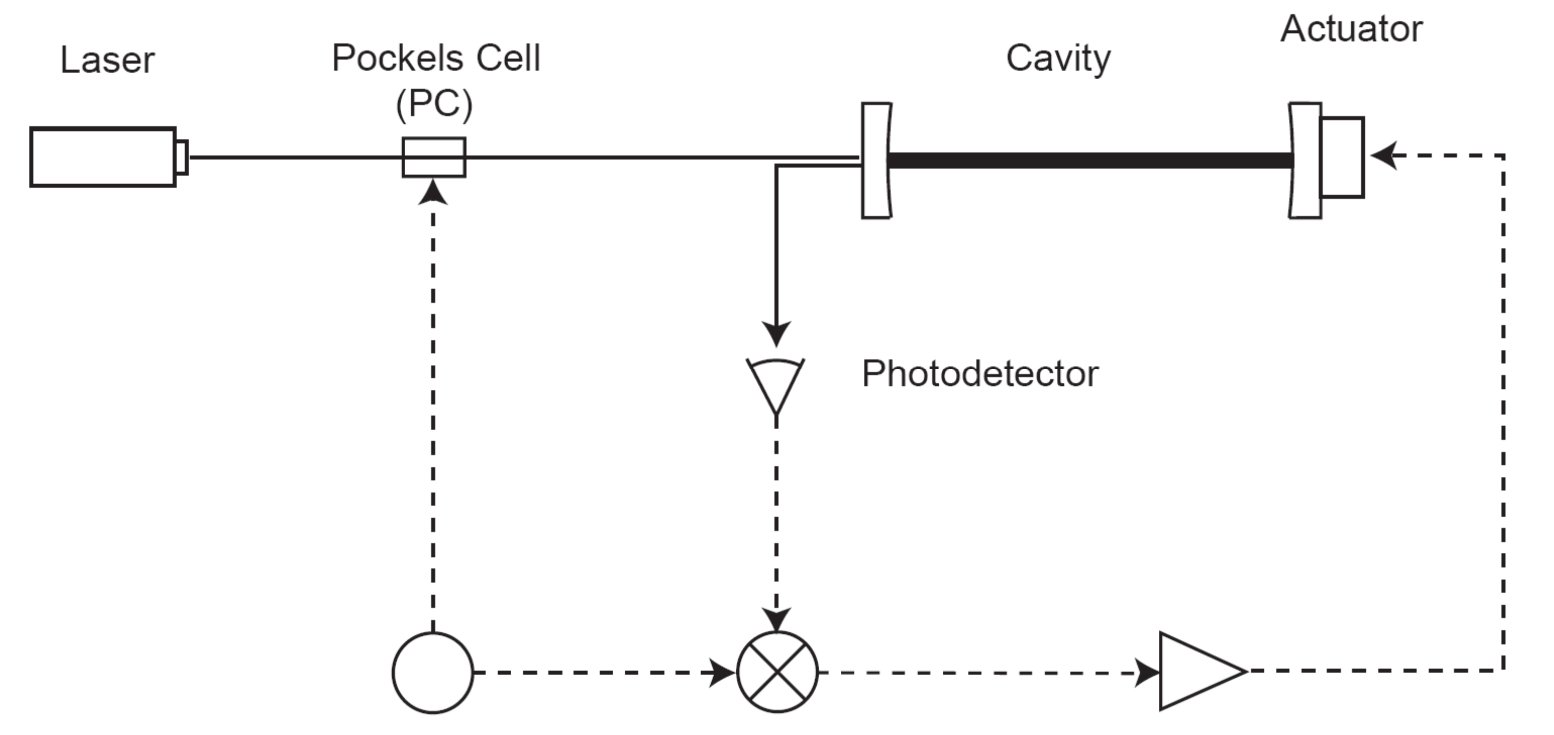
$$E_{out} = A[J_0(\delta)\cos\omega_0 t + J_1(\delta)\{\cos(\omega_0+\Omega)t - \cos(\omega_0-\Omega)t\} \\ +J_2(\delta)\{\cos(\omega_0+2\Omega)t + \cos(\omega_0-2\Omega)t\}..]$$

Amplitude Spectrum of the Output

The output of the phase modulator consists of a series of sidebands spaced by the modulation frequency Ω and whose relative magnitude is given appropriate Bessel function ratio.



Pound-Drever-Hall Locking



$$
\begin{aligned}
E_{inc} &= E_0 e^{i(\omega t + \beta \sin \Omega t)} \\
&\approx E_0 \left[J_0(\beta) + 2 i J_1(\beta) \sin \Omega t \right] e^{i\omega t} \\
&= E_0 \left[J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i(\omega+\Omega)t} - J_1(\beta) e^{i(\omega-\Omega)t} \right].
\end{aligned}
$$

The detected signal

The ratio of the reflected beam to the incident beam gives *F*. For a lossless symmetric cavity we get the following result where Φ represents the phase after one round trip, *i.e.* $2\omega L/c$

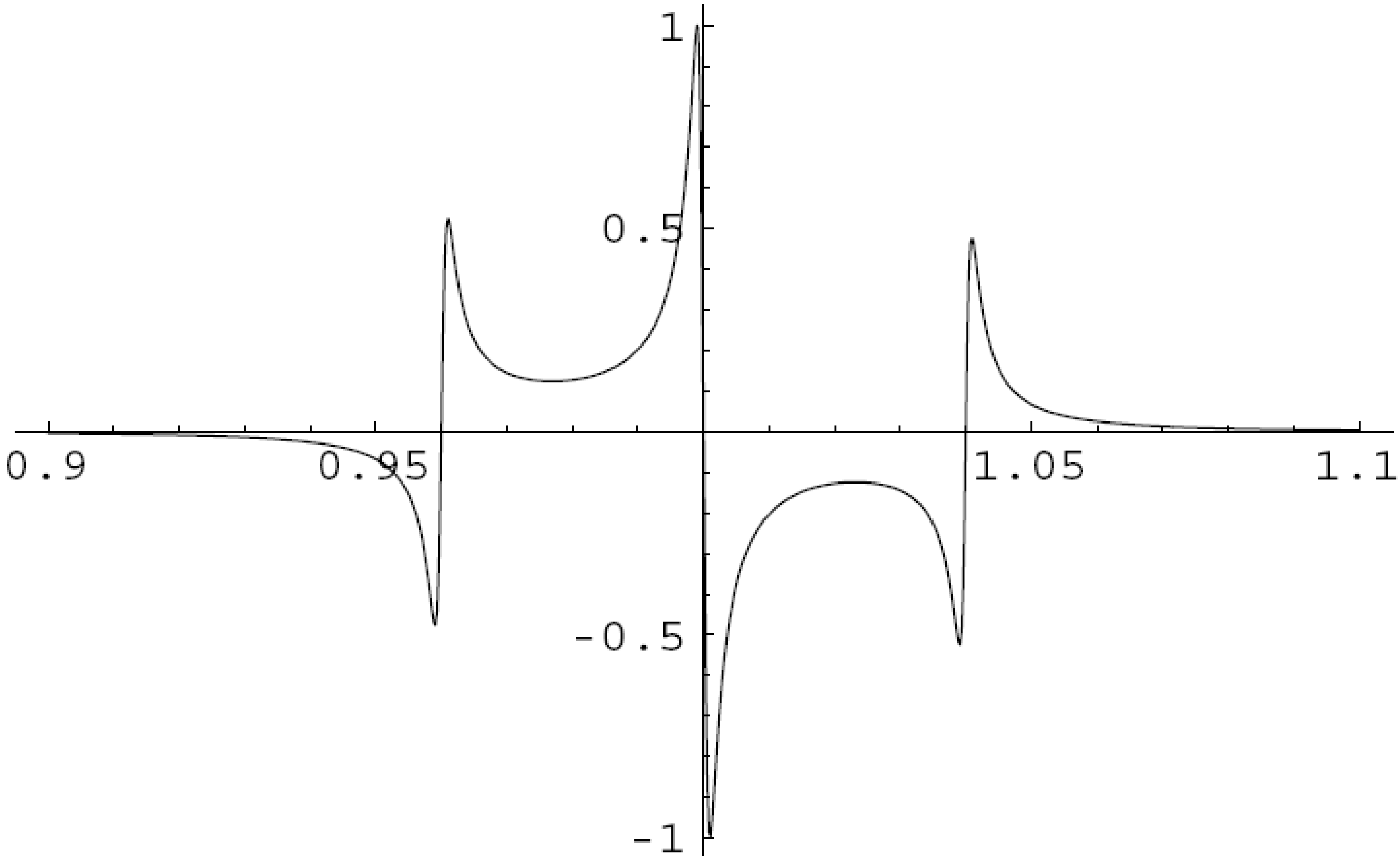
$$F = \frac{r\left(e^{i\phi}-1\right)}{1-r^2 e^{i\phi}}$$

$$E_{ref} = E_0[F(\omega) J_0(\beta) e^{i\omega t} + F(\omega+\Omega) J_1(\beta) e^{i(\omega+\Omega)t} - F(\omega-\Omega) J_1(\beta) e^{i(\omega-\Omega)t}]$$

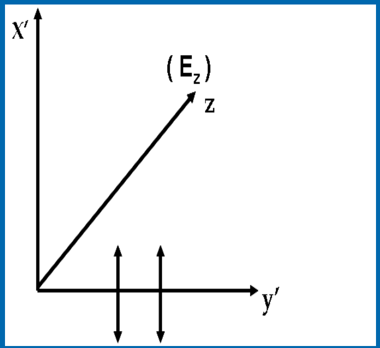
The error signal is obtained by picking out the term going at $\sin \Omega t$

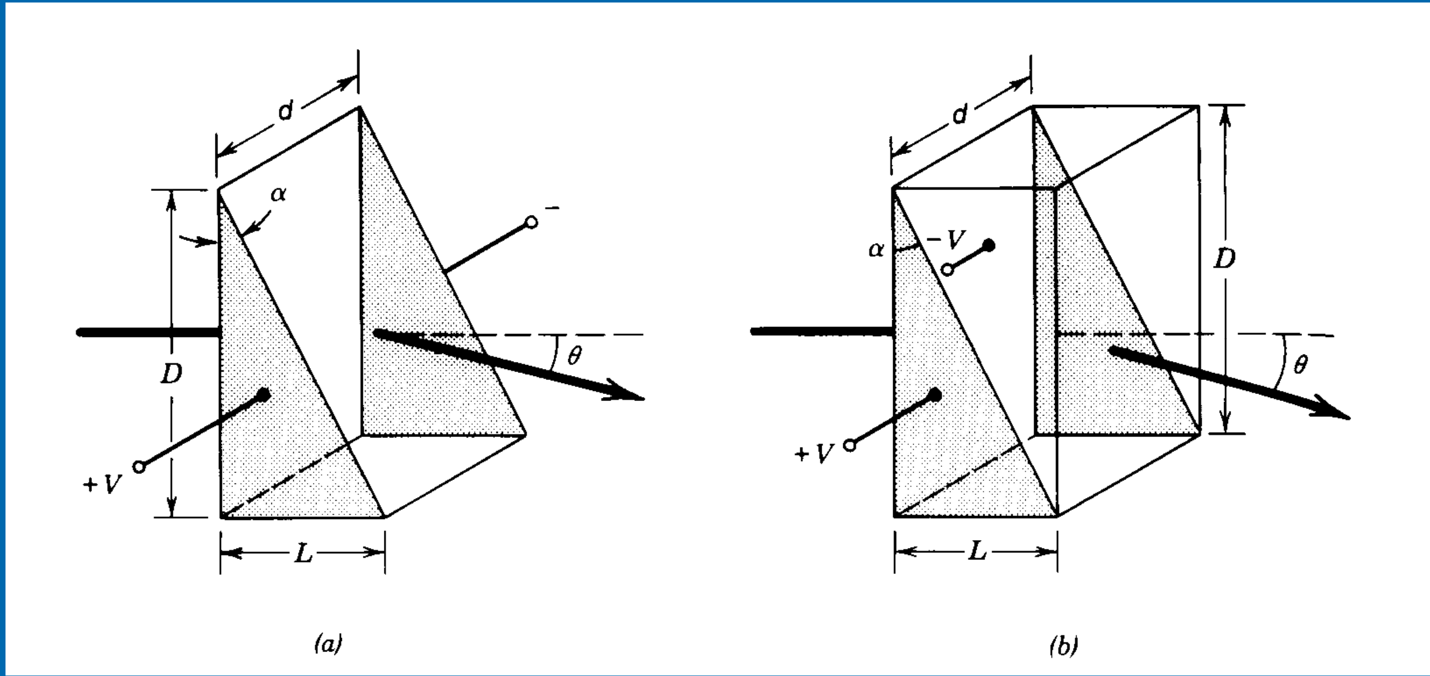
$$\varepsilon = 2\sqrt{P_c P_s}\, \mathrm{Im}\left[F(\omega) F^*(\omega+\Omega) - F^*(\omega) F(\omega-\Omega)\right]$$

The Lock Signal



Electro-optic deflection - 1





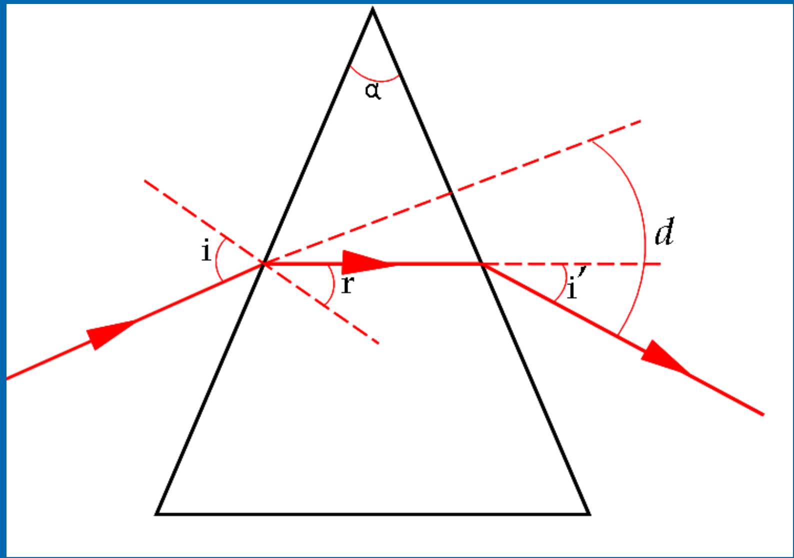
$$\Delta\theta = \alpha\Delta n = -\frac{1}{2}\alpha r n_0^3 E = -\frac{1}{2}\alpha r n_0^3 \frac{V}{d}$$

Electro-optic deflection - 2

$$\alpha = r + r'$$

$$i = nr$$

$$d = (n-1)\alpha \; small\; angle$$

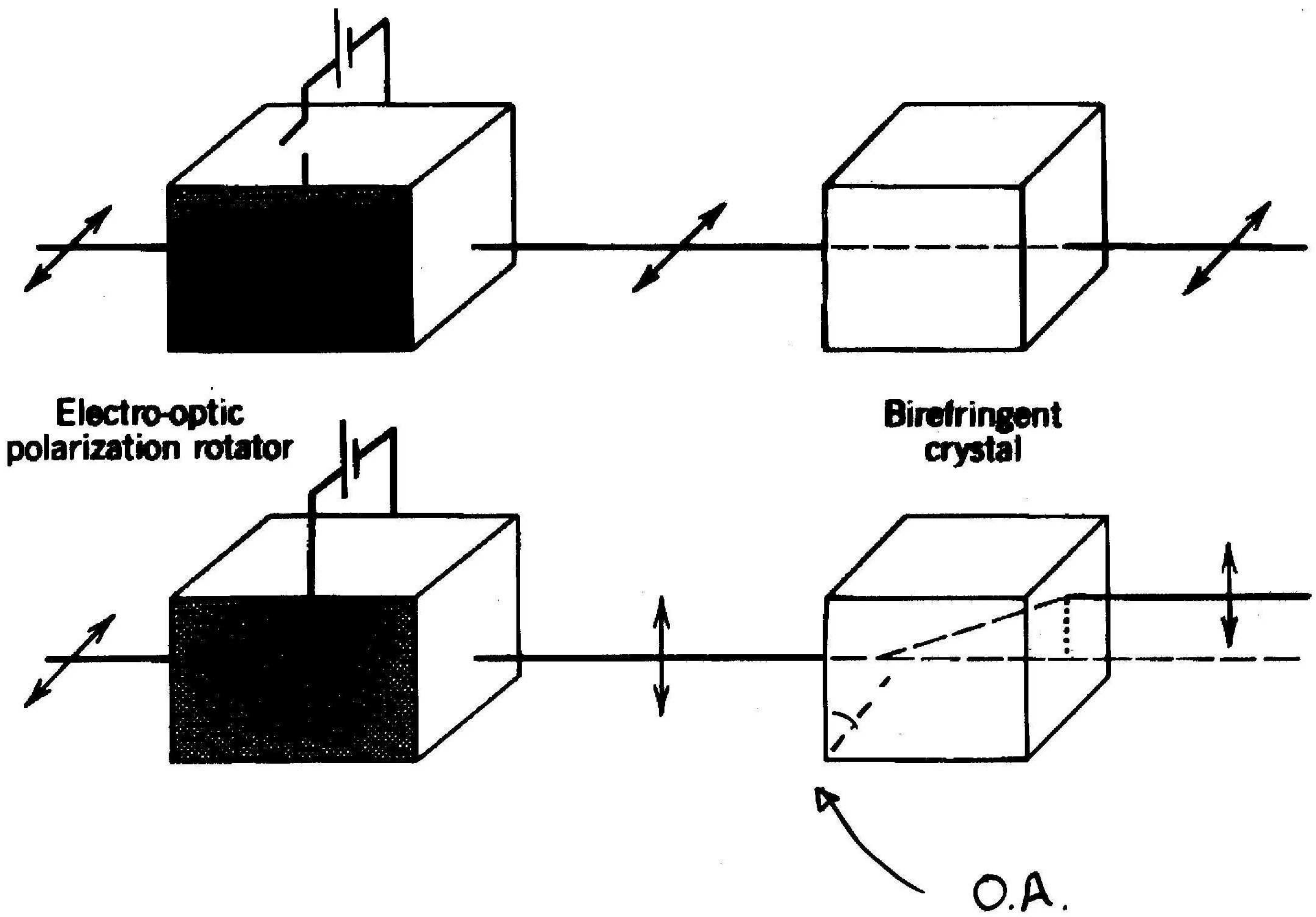


With a given maximum voltage *V* to scan *N* spots requires 2*N* times the half-wave voltage

$$N = \Delta\theta / \delta\theta = \frac{1/2\alpha r n^3 V / d}{\lambda_0 / D} = \frac{V}{2V_\pi}$$

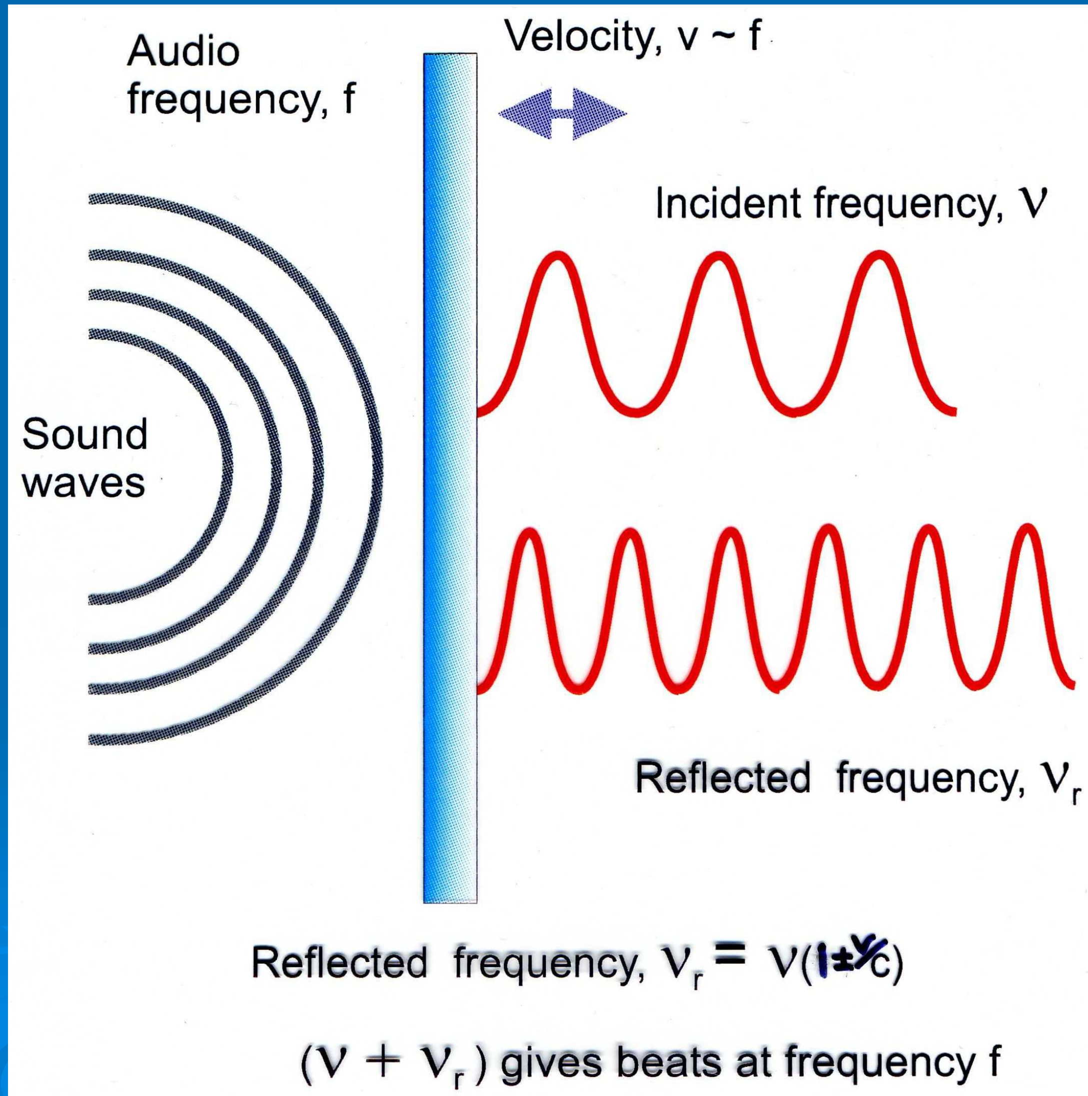
$$\alpha \approx L / D; \quad V_\pi = \left(\frac{d}{L}\right)\left(\frac{\lambda_0}{r\, n^3}\right)$$

Electro-optic deflection - 3

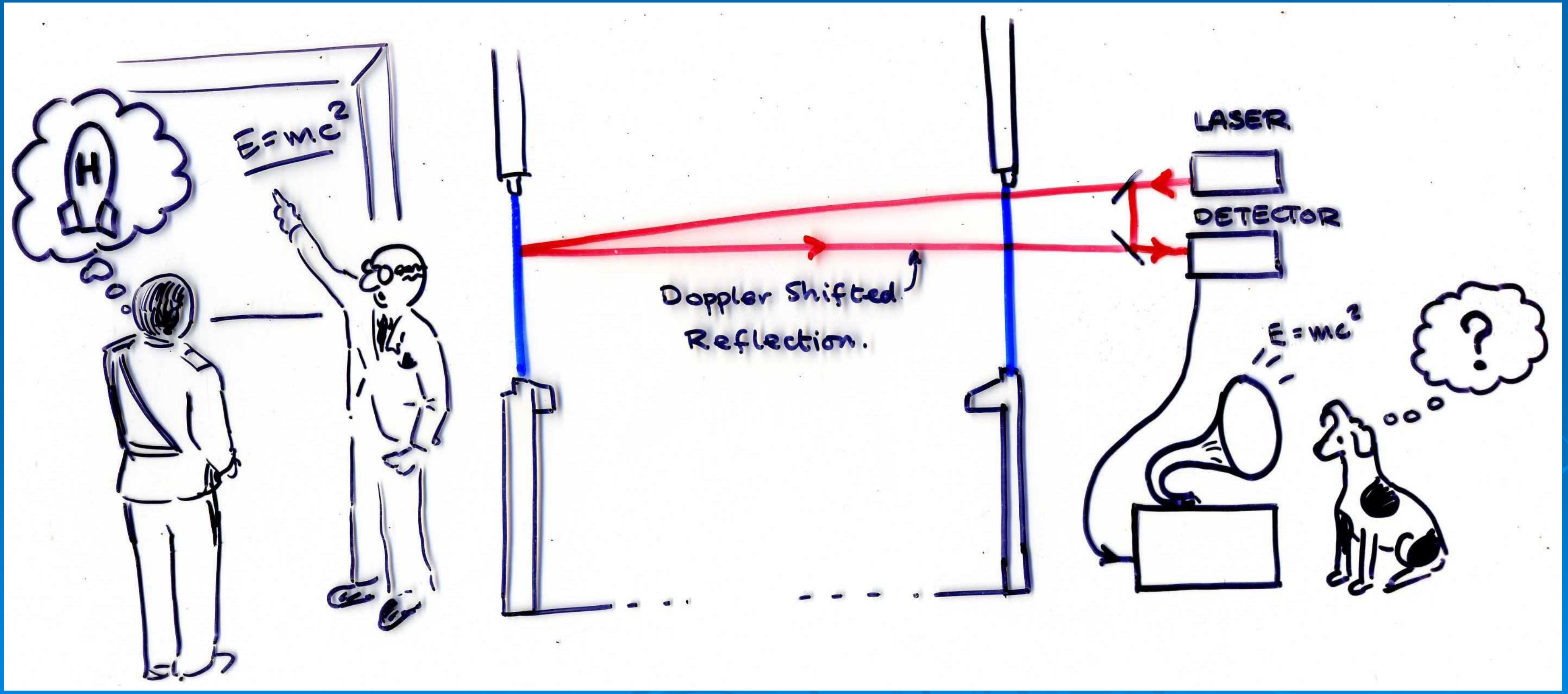


Acousto-optic effects

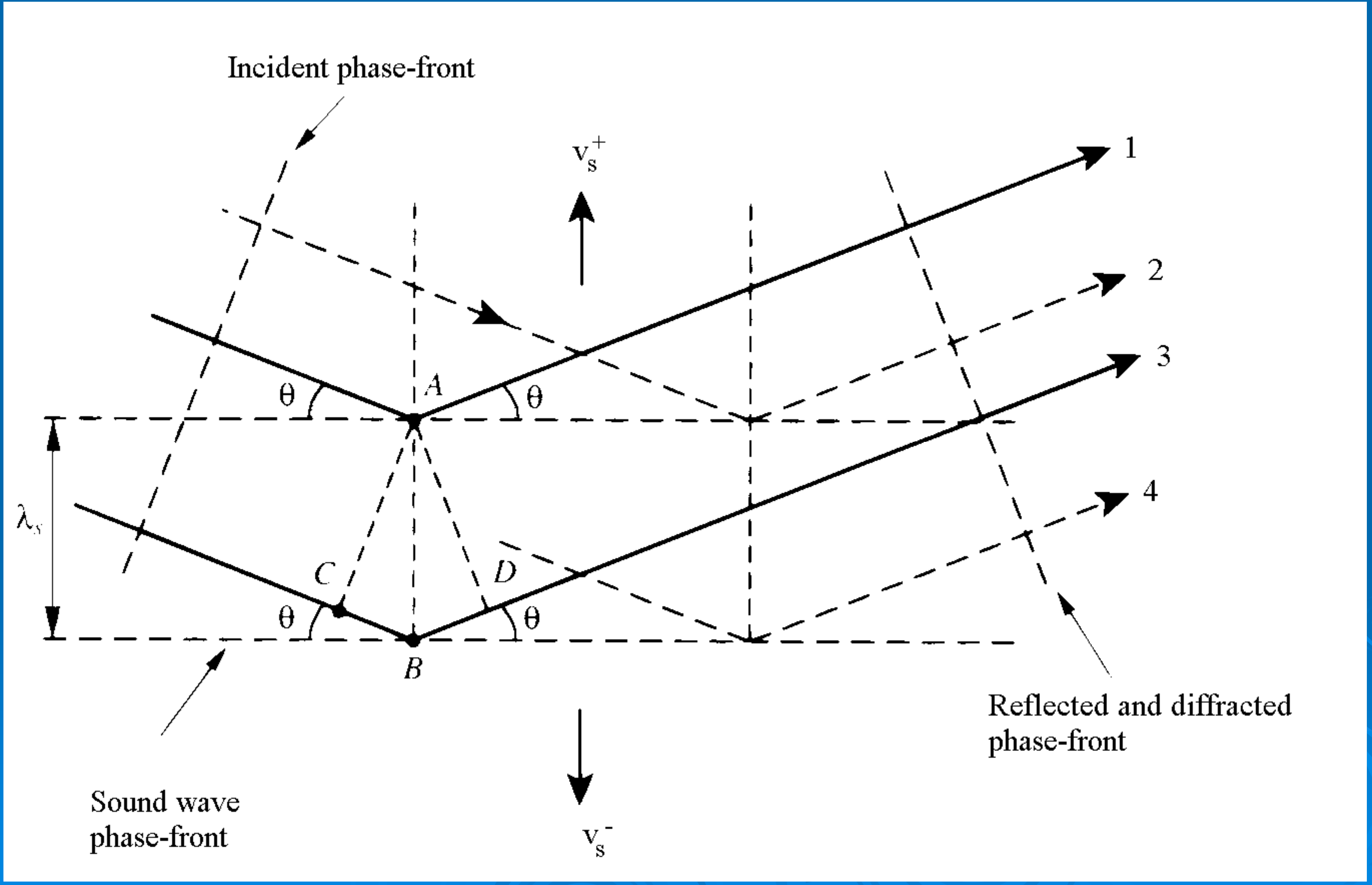
The sound wave modulates the reflector which in turn Doppler shifts the optical wave.



Acousto-optic applications

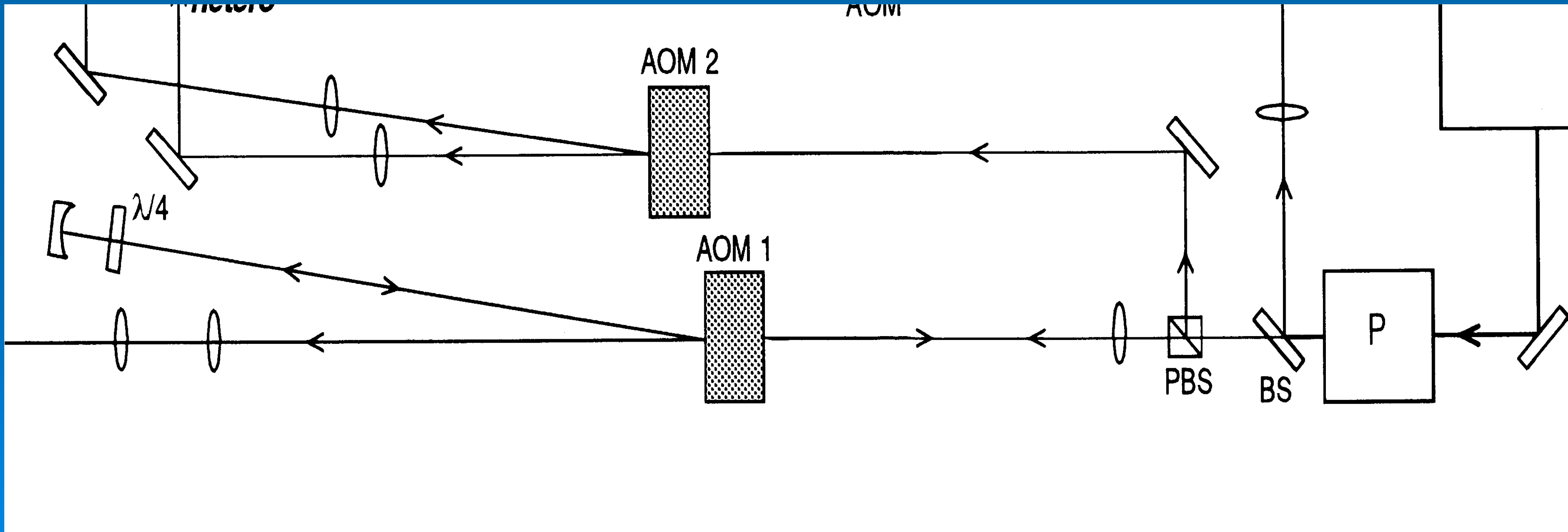


Acousto-optic deflection



Acousto-optic applications

The key feature is to scan the frequency without moving the beam. The double pass through the phase plate rotates the plane of polarisation by 90° allowing the PBS de flect the return beam.



SHG – the susceptibility tensor

For ADP or KDP, the crystal symmetry gives only the following non-zero coefficients for SHG.

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{bmatrix} \begin{bmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_yE_z \\ 2E_xE_z \\ 2E_xE_y \end{bmatrix}$$

Furthermore, $d_{14} = d_{25} \neq d_{36}$

Or if Kleinman's conjecture is invoked all are equal

Second Harmonic Generation

$$\begin{aligned}
\nabla \times (\nabla \times E) &= \nabla(\nabla . E) - \nabla^2 E = \nabla \times \left(-\frac{\partial B}{\partial t}\right) \\
\nabla^2 E &= \frac{\partial}{\partial t}(\nabla \times \mu_0 H) = \frac{\partial}{\partial t}\left(\mu_0 J + \mu_0 \frac{\partial D}{\partial t}\right) \\
&= \mu_0 \frac{\partial^2}{\partial t^2}(\varepsilon_0 E + P_L + P_{NL}) = \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2 E}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}
\end{aligned}$$

$$\nabla^2 E - \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

SHG: more maths

Now insert the following and take a 1D plane wave:

$$P_{NL} = \varepsilon_0 \frac{\chi}{2} E_\omega E_\omega; \quad E_\omega = A_\omega e^{ik_\omega z}$$

where $A_\omega = A_0(z)\exp(-i\omega t)$ and similarly for 2ω

$$\begin{aligned} \frac{d^2 E_{2\omega}}{dz^2} + \frac{(2\omega)^2 n^2}{c^2} E_{2\omega} &= -\mu_0 (2\omega)^2 P_{2\omega} \\ -k_{2\omega}^2 A_{2\omega} + 2ik_{2\omega}\frac{dA_{2\omega}}{dz} + \left(\frac{n_{2\omega}2\omega}{c^2}\right)^2 A_{2\omega} &= -\mu_0 (2\omega)^2 \varepsilon_0 \frac{\chi}{2} A_\omega^2 \end{aligned}$$

SHG: and yet more..

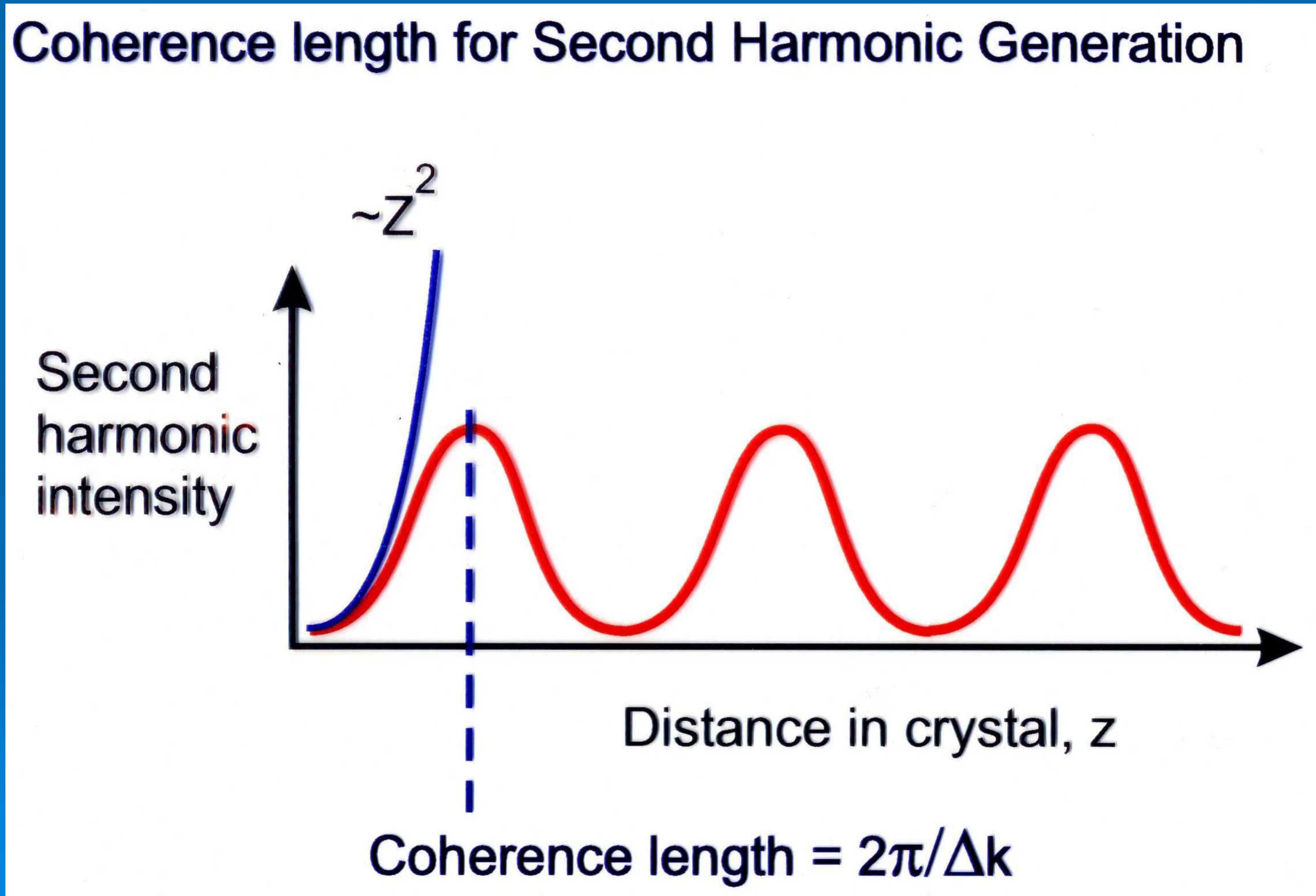
Now assuming a small variation of *A* with *z*, and using $\Delta k = 2k_\omega - k_{2\omega}$

$$\frac{i\left(2\omega\right)^2 A_\omega^2 \chi \exp\left(i\Delta kz\right)}{2c^2 2k_{2\omega}} = \frac{i\left(2\omega\frac{1}{2}\chi^{SHG}\right) A_\omega^2}{2n_{2\omega}c} \exp\left(i\Delta kz\right) = \frac{dA_{2\omega}}{dz}$$

$$\Longrightarrow \int_0^z \exp\left(i\Delta kz\right) dz \Longrightarrow sinc\ \{\}$$

$$I^{2\omega} = \frac{(2\omega)^2(\frac{1}{2}\chi^{SHG})^2}{2n^{2\omega}(n^\omega)^2 c^3 \varepsilon_0}(I^\omega)^2 \left\{\frac{\sin(\Delta kz/2)}{\Delta kz/2}\right\}^2 z^2$$

SHG: so finally…



Phase-matching