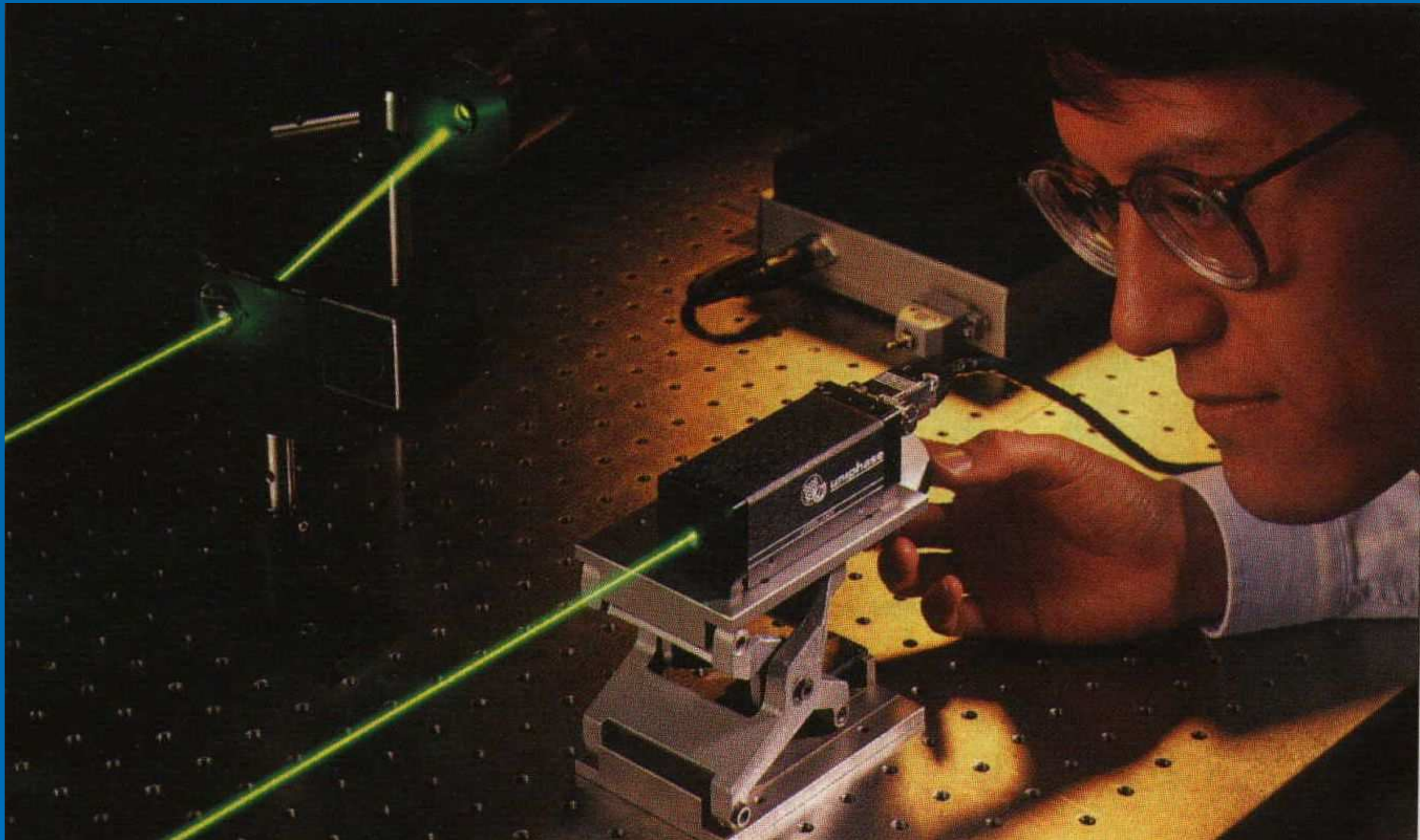
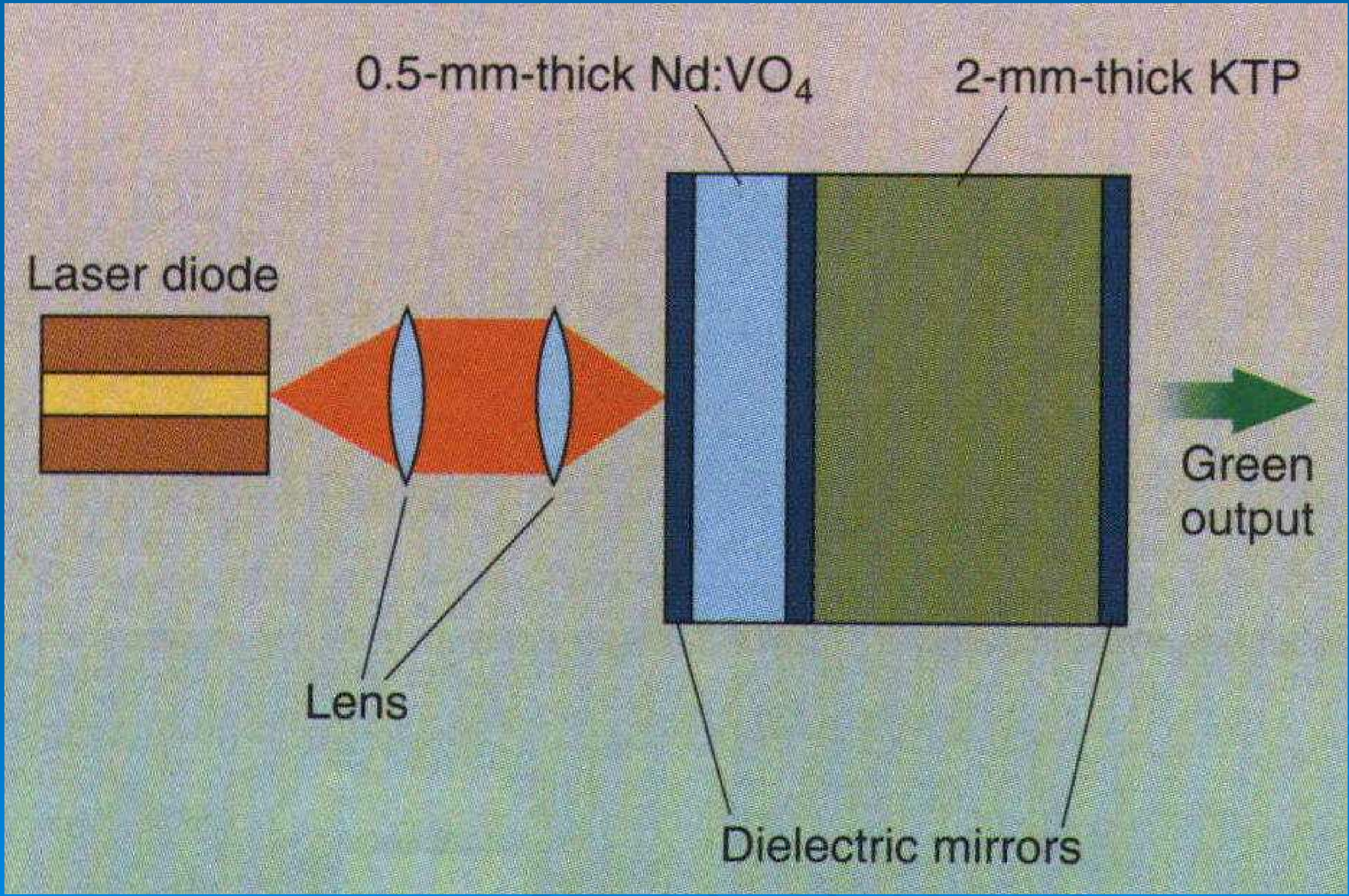
Non-linear Effects



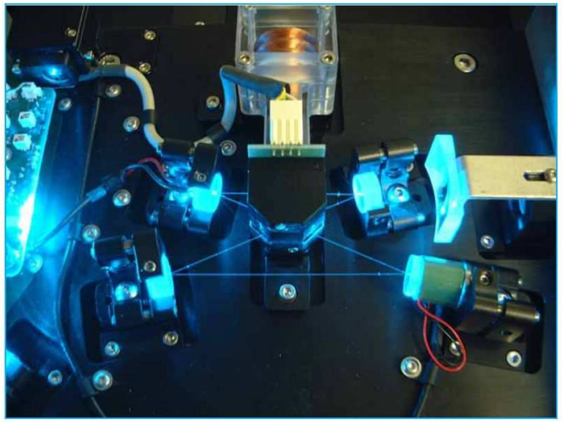
100mW of cw output from a package 3.8cm x 3.8 cm x 10cm.The device consists of a chip of 0.5mm of Nd:YV0$_4$ in contact with a 2mm KTP crystal; 500mW of laser output at 809nm is used to pump the device.

And inside the box…



Doubling and doubling again!

Optical Excitation Setup



972 nm to 486 nm frequency doubler

$$n = 2.17 \times 10^{-7} N\pi \frac{P^2}{\lambda\, df} \arctan\left(\frac{\lambda l}{2\pi W_0^2}\right)$$

n = number of metastables

N = number of ground state atoms

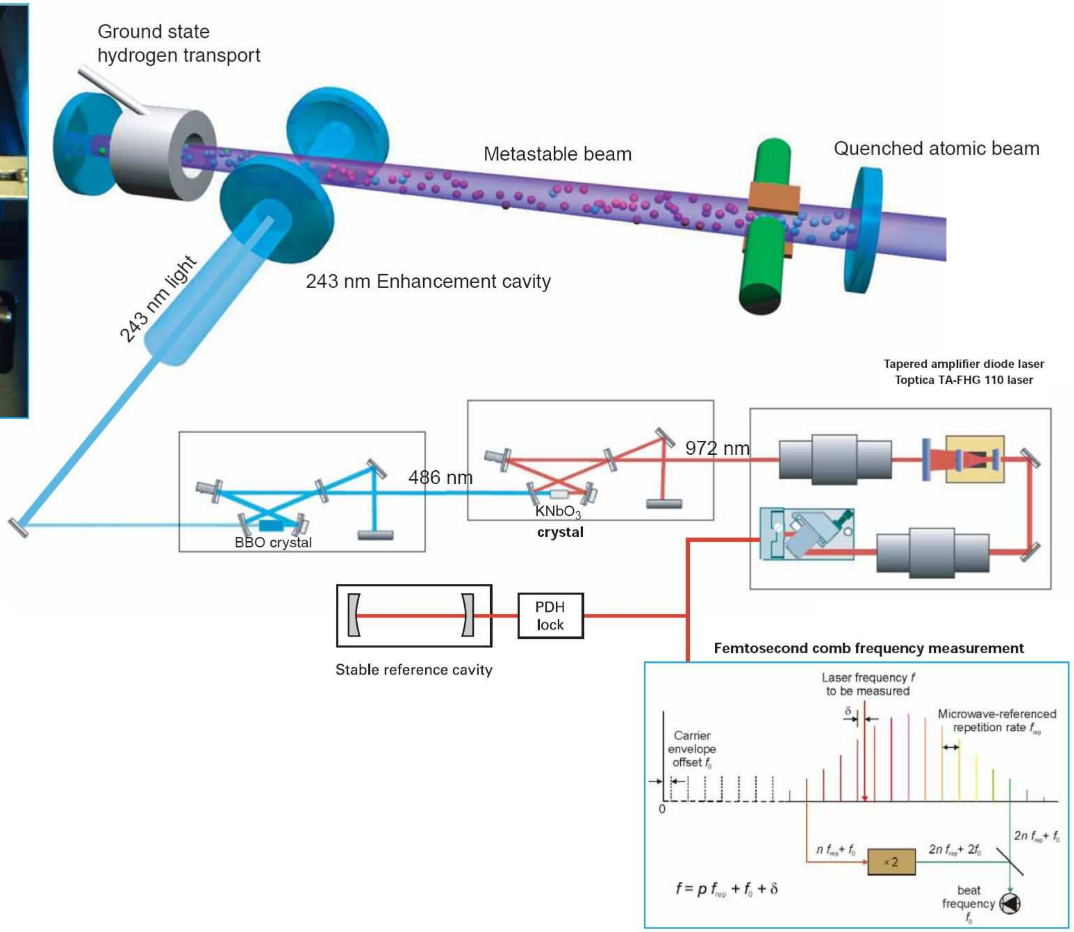
P = laser circulating power at 243 nm

λ = laser wavelength (243 nm)

df = transition broadening

l = hydrogen beam width

W_0 = laser beam waist

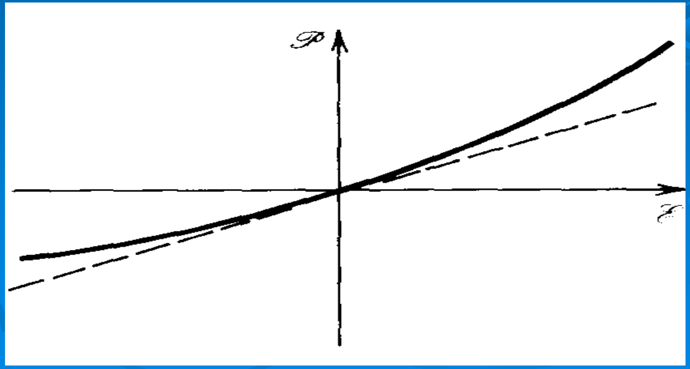


Non-linear Optics

- The polarisation, *P*, can be described in terms of the susceptibility *tensor*, χ. We can now include any non-linear response of the medium as shown below.
- Note that in general the electric field and the polarisation need NOT be collinear.

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E}(1 + \chi)$$

$$P_i = \varepsilon_0 \sum \chi_{ij} E_j$$



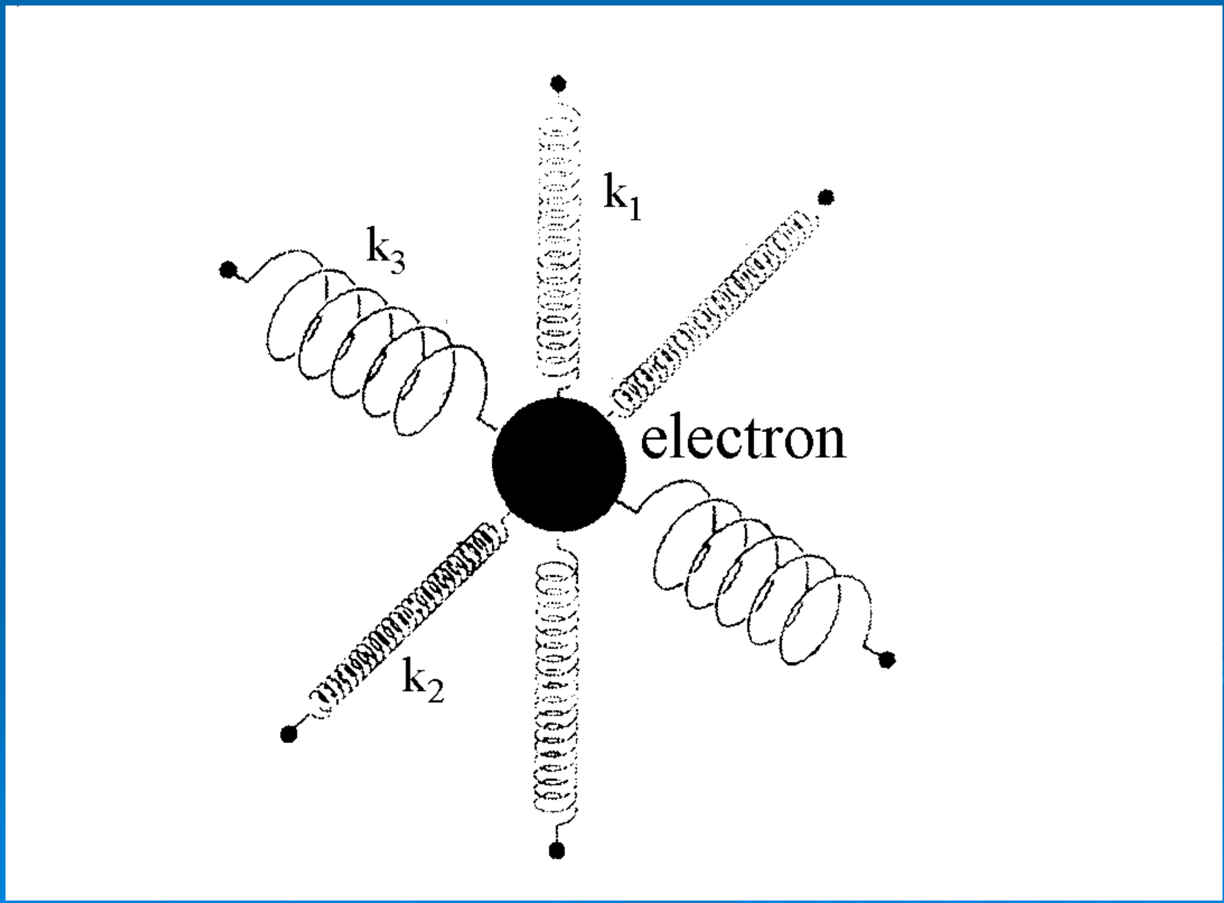
Non-linear Effects:
(Classification by order)

$$P_i^n(t) = \varepsilon_0 \chi^{(n)} \left(-\omega_\sigma; \omega_1, \omega_2, \omega_3\right) E_j E_k E_\ell \ldots \exp\left\{-i\omega_\sigma t\right\}$$

Optical Effect	**Order of χ**	**Indices**	**Examples**
Linear absorption	1	$-\omega; \omega$	Laser absorption at low intensity
Pockels effect	2	$-\omega; 0, \omega$	Electro-optic modulators
Second harmonic generation	2	$-2\omega; \omega, \omega$	Frequency doubling of laser light
Sum & difference generation	2	$-\omega_3; \omega_1, \omega_2$	Generation of new frequencies: fixed + tunable
D.C. Kerr effect	3	$-\omega; 0, 0, \omega$	Electro-optic devices
Third harmonic generation	3	$-3\omega; \omega, \omega, \omega$	
Four-wave mixing	3	$-\omega_4; \omega_1, \omega_2, \omega_3$	Holography
Optical Kerr effect	3	$-\omega; \omega, -\omega, \omega$	Intensity-dependent refractive index
Two-photon absorption	3	$-\omega; -\omega, \omega, \omega$	Doppler-free spectroscopy

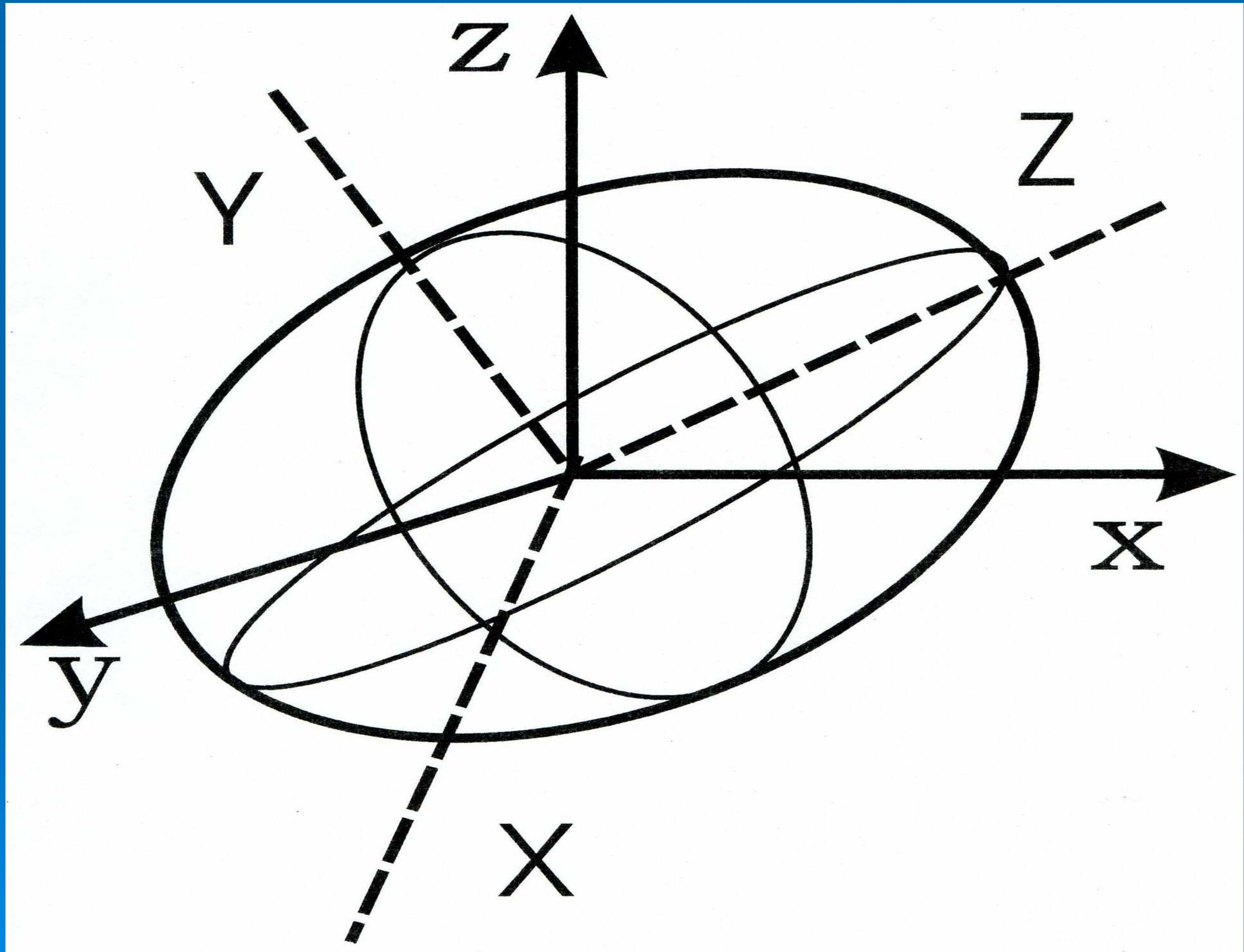
Anisotropic binding of an electron in a crystal

The springs have different stiffness for different directions of the electron’s displacement from its equilibrium position within the lattice.



The polarisation, and therefore the refractive index, will be different in different directions

Index ellipsoid and the principal axes



The Permittivity Tensor

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \varepsilon_0 \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

This represents a 3D ellipsoidal surface with axes x, y, z

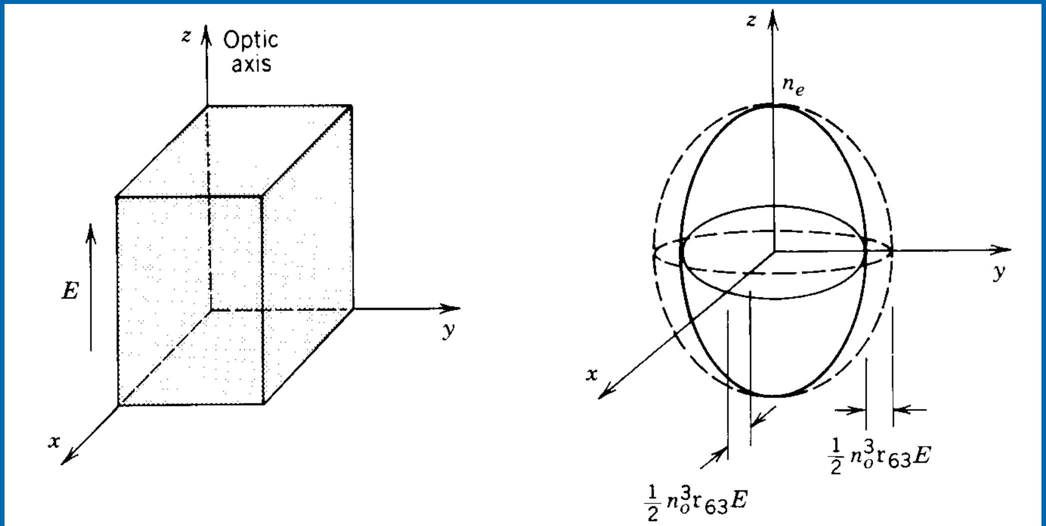
With respect to the symmetry axes XYZ this becomes (by suitable rotation):

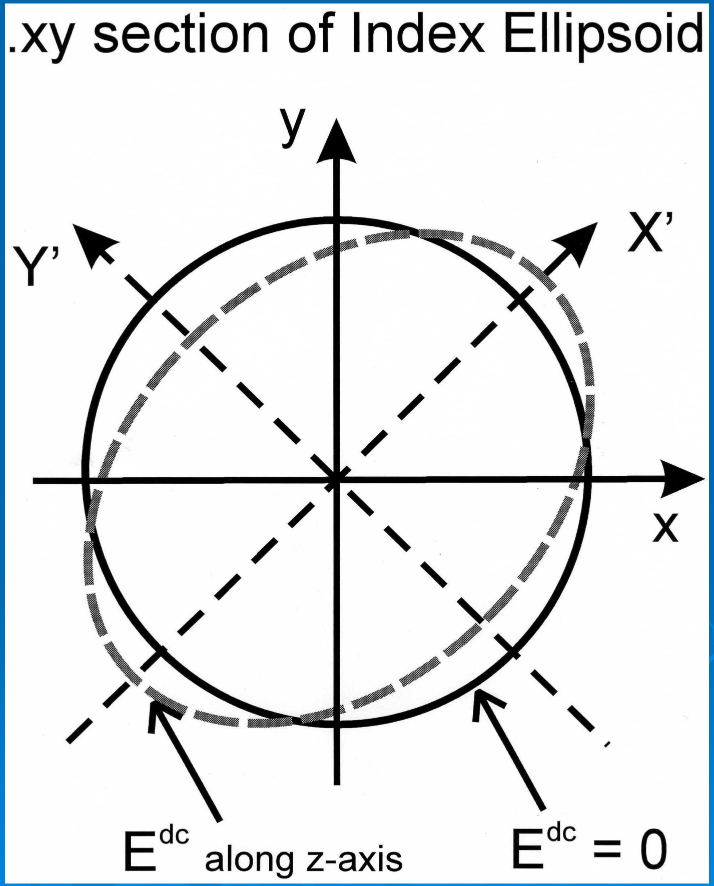
$$\begin{pmatrix} D_X \\ D_Y \\ D_Z \end{pmatrix} = \varepsilon_0 \begin{pmatrix} \varepsilon_X & 0 & 0 \\ 0 & \varepsilon_Y & 0 \\ 0 & 0 & \varepsilon_Z \end{pmatrix} \begin{pmatrix} E_X \\ E_Y \\ E_Z \end{pmatrix}$$

The equation of the *index ellipsoid* is thus,

$$\boxed{\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} = 1}$$

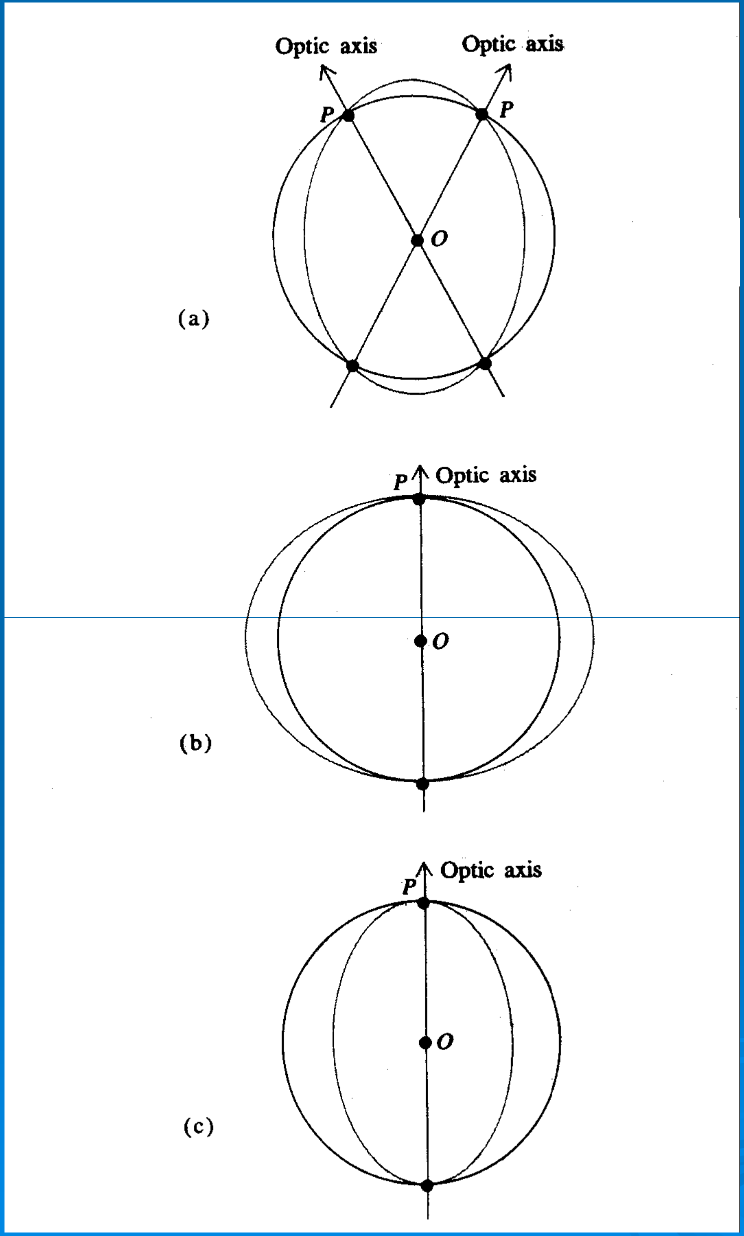
Electro-optic effect





Distortion of index ellipsoid caused by
The application of an electric field

Uniaxial or Biaxial?



(a)Biaxial crystal – two optical axes.

(b) Positive uniaxial crystal – one optical axis $n_e \geq n_o$

(c) Negative uniaxial crystal – one optical axis $n_e \leq n_o$

The Index Ellipsoid

$$\eta = \varepsilon_0/\varepsilon = 1/n^2$$

$$\sum_{ij} \eta_{ij} x_i x_j = 1$$

3D ellipsoid surface is given by:

$$\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} + \frac{2yz}{n_4^2} + \frac{2xz}{n_5^2} + \frac{2xy}{n_6^2} = 1$$

Linear Electro-optic Tensor

The *r*-coefficients are related to the crystal symmetry

$$\Delta\left(\frac{1}{n^2}\right) = \sum r_{ijk} E_k^0$$

The 27-elements reduce to 18 because of invariance w.r.t. *i,j* interchange

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \begin{bmatrix} E_1^0 \\ E_2^0 \\ E_3^0 \end{bmatrix}$$

The Electro-optic Ellipsoid

Taylor expand the refractive index in the electric field

$$n(E) = n_0 + a_1 E + \frac{1}{2} a_2 E^2 \ldots$$

And remembering,

$$\eta = \varepsilon_0 / \varepsilon = 1/n^2$$

So,

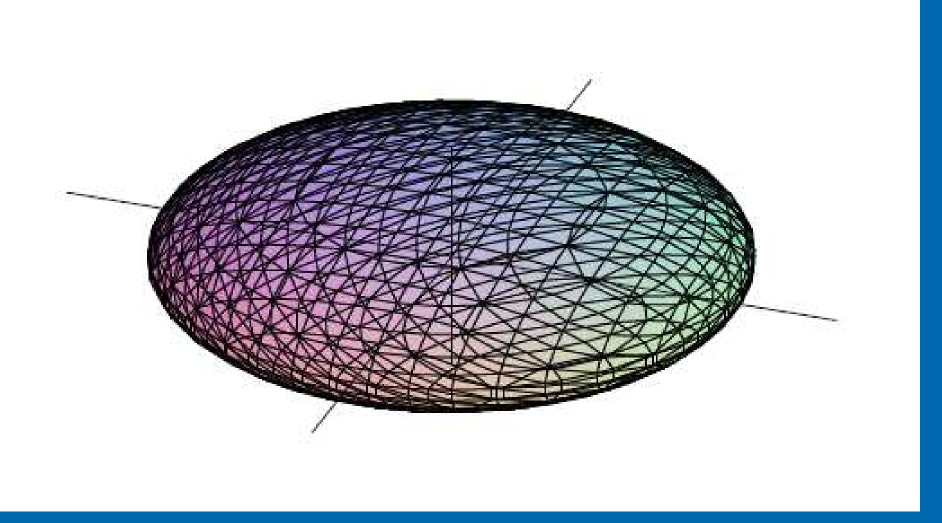
$$\Delta\eta = (d\eta/dn)\Delta n = (-2/n^3)(-\tfrac{1}{2} r n^3 E - \tfrac{1}{2} s n^3 E^2 ..).$$

$$n(E) = n_0 - \frac{1}{2} n^3 r E - \frac{1}{2} n^3 s E^2 + \ldots$$

With the identities,

$$r = -2a_1/n^3 \qquad s = -a_2/n^3$$

Index Ellipsoid with Electric Field



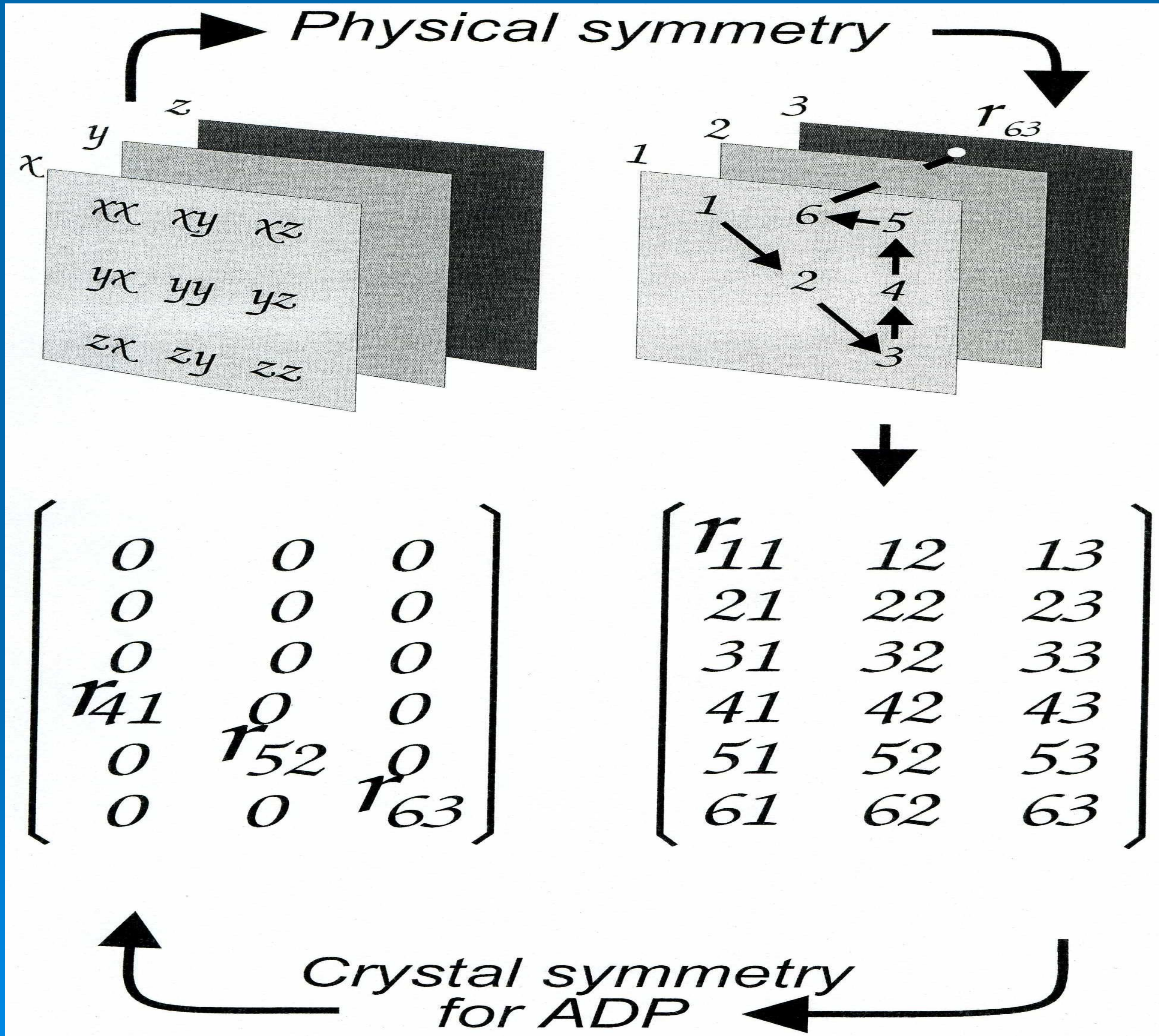
$$\eta_{ij}(E) = \varepsilon_0/\varepsilon = 1/n^2 = \eta_{ij} + \sum_k r_{ijk}E_k + \sum_{k,\ell} s_{ijk\ell}E_kE_\ell ....$$

$$\sum_{ij} \eta_{ij} x_i x_j = 1$$

$$n(E) = n_0 + a_1 E + \tfrac{1}{2} a_2 E^2 + ...$$

$$n(E) = n_0 - \tfrac{1}{2} n^3 r E - \tfrac{1}{2} n^3 s E^2 + ...$$

Electro-optic tensor



Case of ADP and isomorphs

With a field applied in the z-direction the ellipsoid is distorted in the xy-plane

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_e^2} + 2r_{63}xyE_z^0 = 1$$

Rotation by 45° about the z-axis transforms to the (x´, y´, z´) co-ordinate system

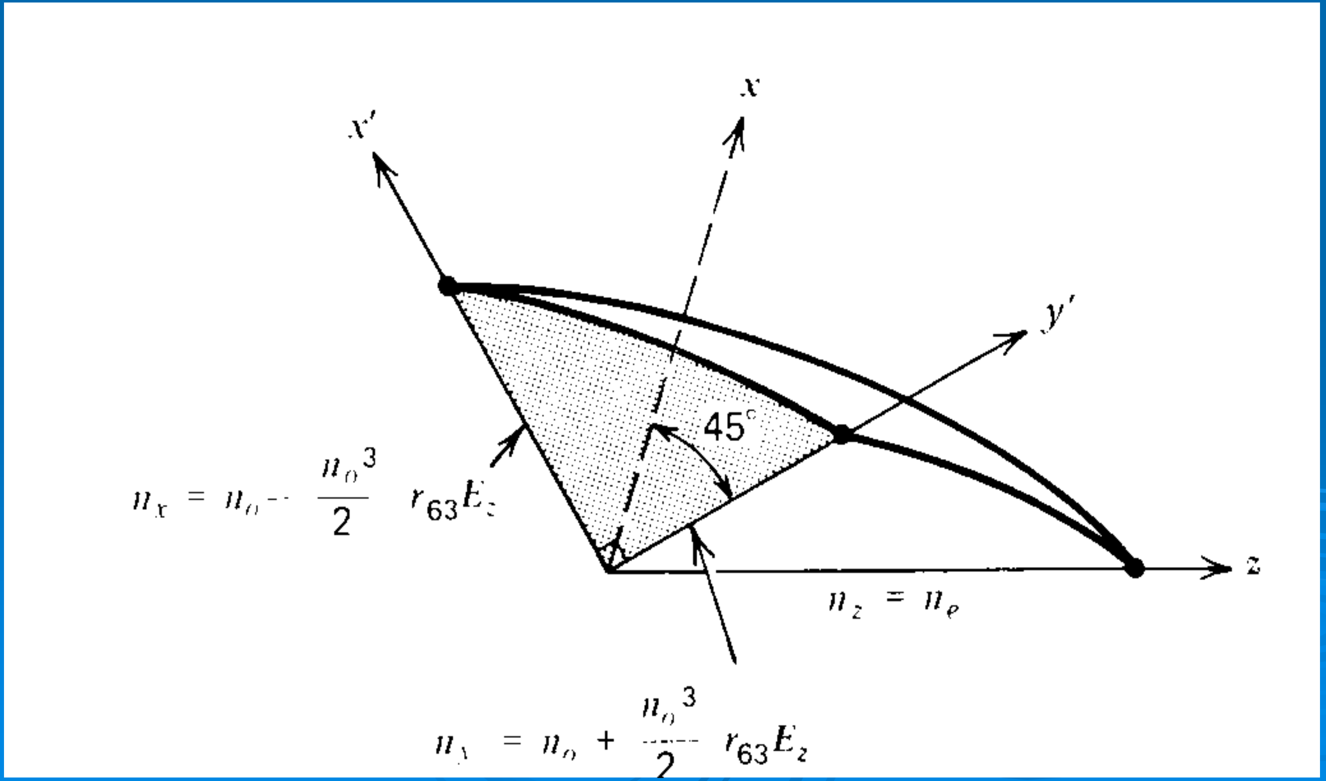
$$\left(\frac{1}{n_0^2} + r_{63}E_z^0\right)x'^2 + \left(\frac{1}{n_0^2} - r_{63}E_z^0\right)y'^2 + \frac{z'^2}{n_e^2} = 1$$

The transformed axes

So, given, $r_{63}E_z^0 \ << \ n_o^{-2}$

we can write, $n_{x'}^{-2} \ = \ n_o^{-2} + r_{63}E_z^0$

i.e., $n_{x'} = n_0(1+n_0^2 r_{63} E_z^0)^{-1/2}$



Variable phase-plate

With light polarised along the original x-direction

$$\Delta n = |n_{x'} - n_{y'}| = n_0^3 r_{63} E_z^0$$

For a half-wave plate:

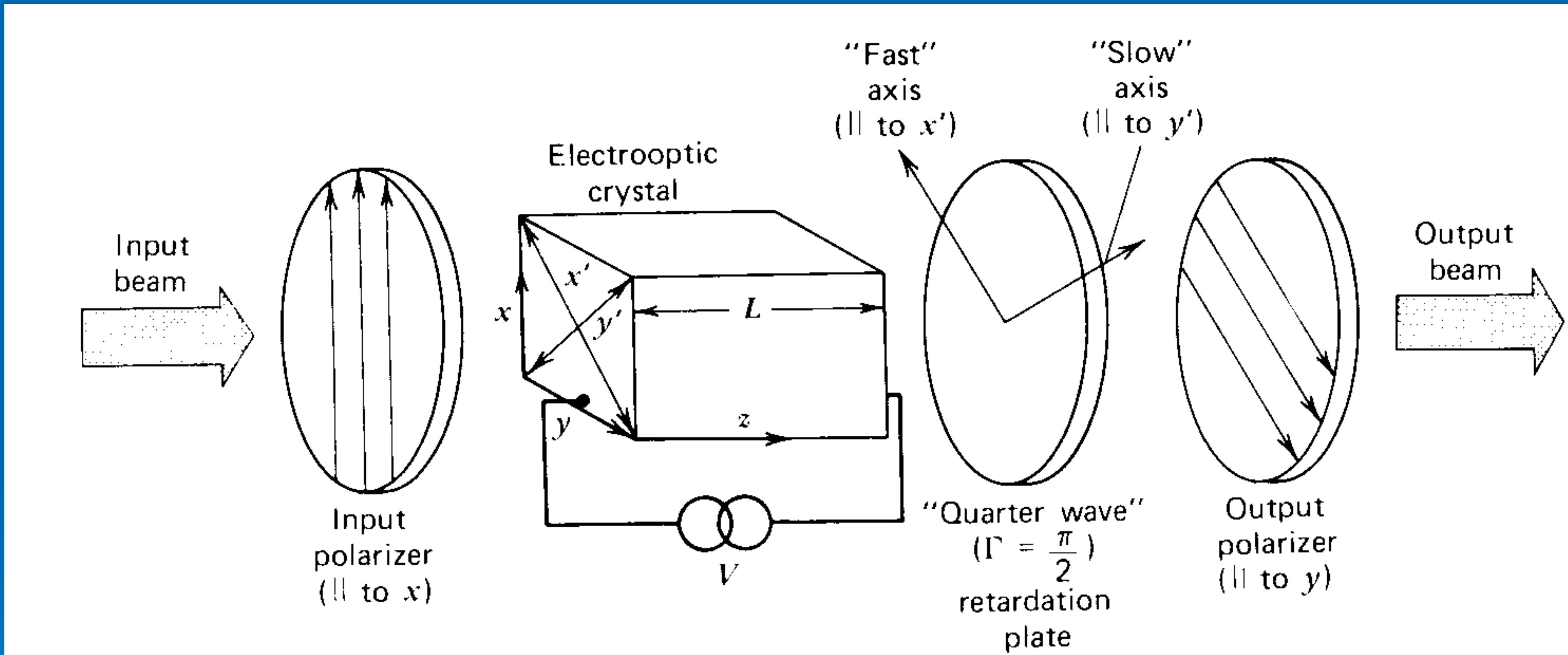
$$\phi = \frac{2\pi}{\lambda} \Delta n d = \pi$$

Giving a half-wave Voltage:

$$V_\pi = \frac{\lambda}{2 n_0^3 r_{63}}$$

(Note: the field & propagation directions are the same here!)

Amplitude modulation



The susceptibility tensor

From Maxwells eqns, a plane, monochromatic wave satisfies

$$\mathbf{k}\times(\mathbf{k}\times\mathbf{E})+\frac{\omega^2}{c^2}\mathbf{E}=-\frac{\omega^2}{c^2}\chi\mathbf{E}$$

$$\left(-k_y^2-k_z^2+\frac{\omega^2}{c^2}\right)E_x+k_xk_yE_y+k_xk_zE_z=-\frac{\omega^2}{c^2}\chi_{11}E_x$$

Propagate the wave along the principal z- axis so that now

$$\begin{aligned}\left(-k^2+\frac{\omega^2}{c^2}\right)E_x &= -\frac{\omega^2}{c^2}\chi_{11}E_x\\ \left(-k^2+\frac{\omega^2}{c^2}\right)E_y &= -\frac{\omega^2}{c^2}\chi_{22}E_y\\ \frac{\omega^2}{c^2}E_z &= -\frac{\omega^2}{c^2}\chi_{33}E_z\end{aligned}$$

The wave-vector surface

$E_z = 0$; the wave is transverse, so

$$k = \frac{\omega}{c}\sqrt{1+\chi_{11}} = \frac{\omega}{c}\sqrt{K_{11}} = \frac{\omega}{c}n_1$$
$$k = \frac{\omega}{c}\sqrt{1+\chi_{22}} = \frac{\omega}{c}\sqrt{K_{22}} = \frac{\omega}{c}n_2$$

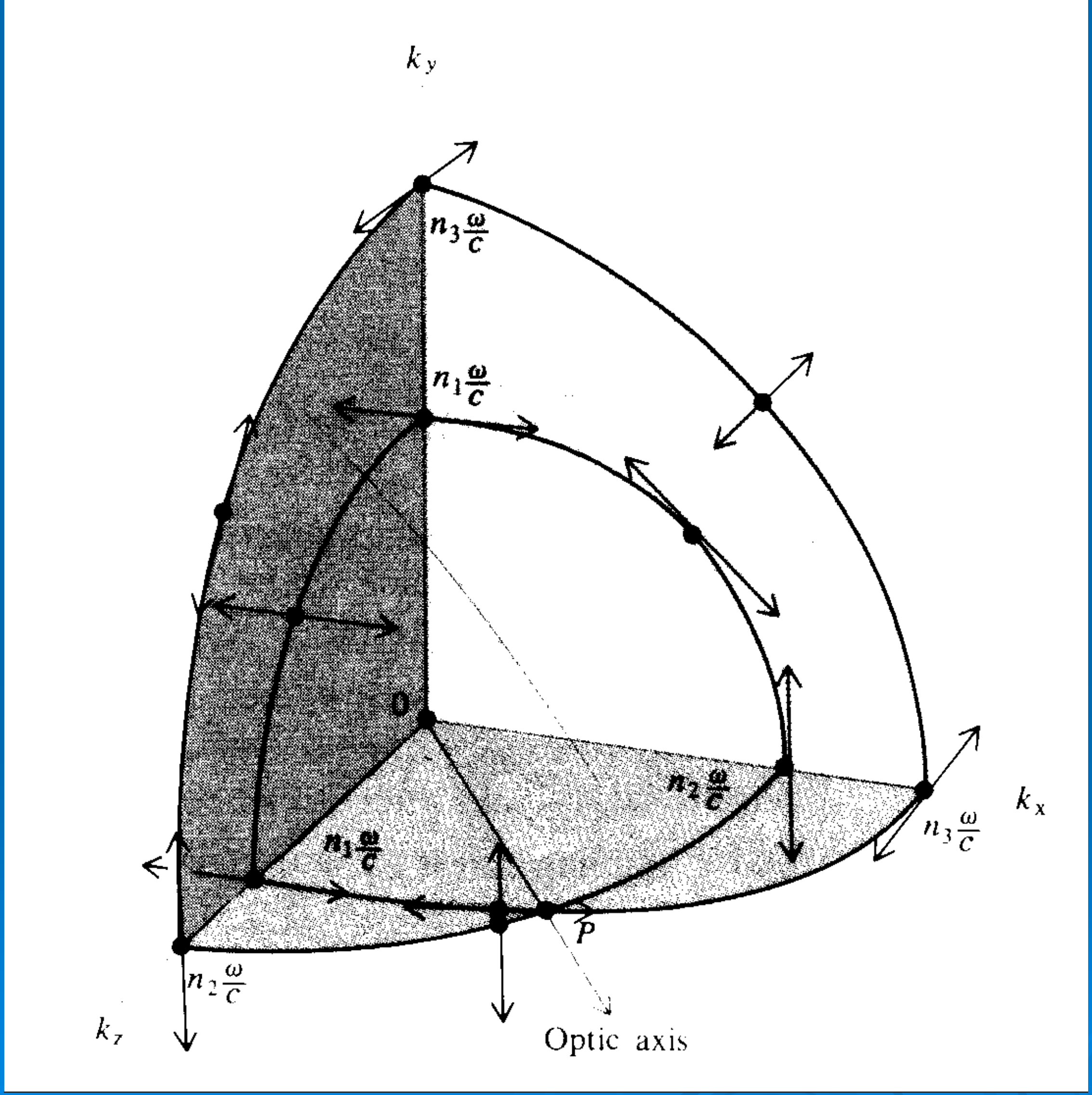
For non-trivial solutions

$$\begin{vmatrix} (\frac{n_1\omega}{c})^2 - k_y^2 - k_z^2 & k_x k_y & k_x k_z \\ k_y k_x & (\frac{n_2\omega}{c})^2 - k_x^2 - k_z^2 & k_y k_z \\ k_z k_x & k_z k_y & (\frac{n_3\omega}{c})^2 - k_x^2 - k_y^2 \end{vmatrix} = 0$$

which gives the equation for a circle and an ellipse

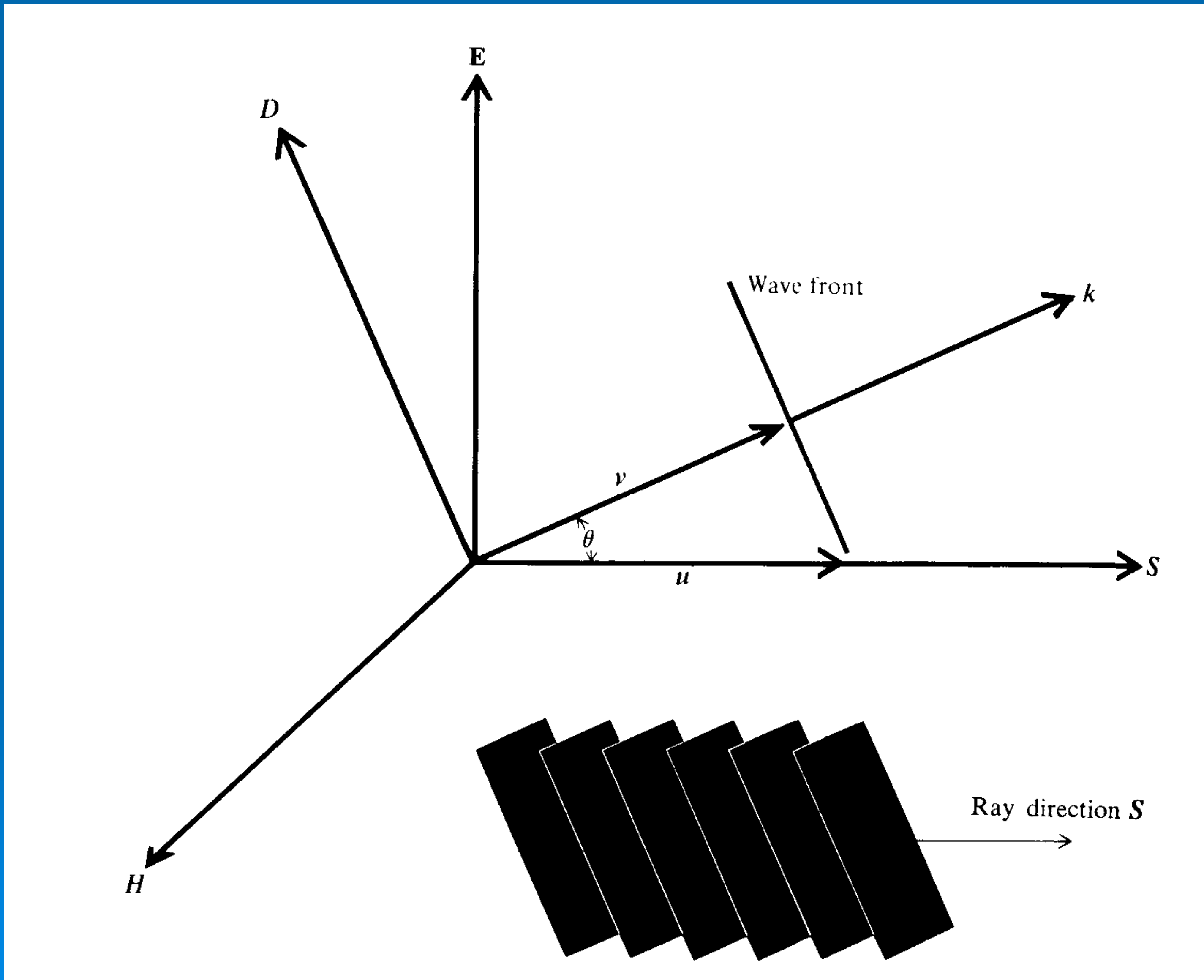
$$k_x^2 + k_y^2 = \left(\frac{n_3\omega}{c}\right)^2$$
$$\frac{k_x^2}{(n_2\omega/c)^2} + \frac{k_y^2}{(n_1\omega/c)^2} = 1$$

The wave-vector surface



The intercept of the k-surface with each plane xy, xz & yz Consists of one circle and one ellipse. The surface is double suggesting there are two possible values for *k* for any direction of the vector **k**. There are two phase velocities corresponding to two orthogonal polarisations. At the point P the two values are equal; this is the optical axis.

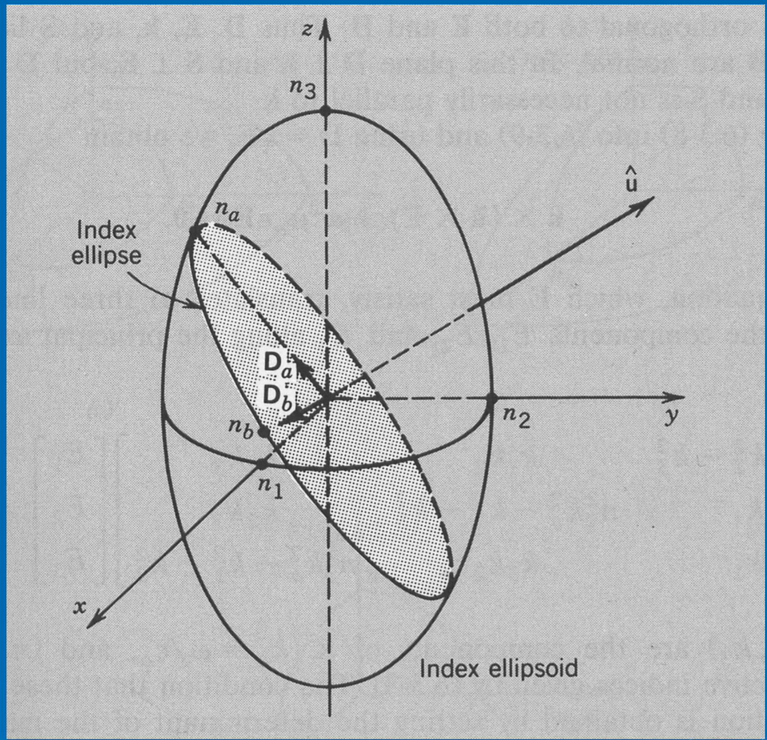
Poynting's vector- S & wavevector- k

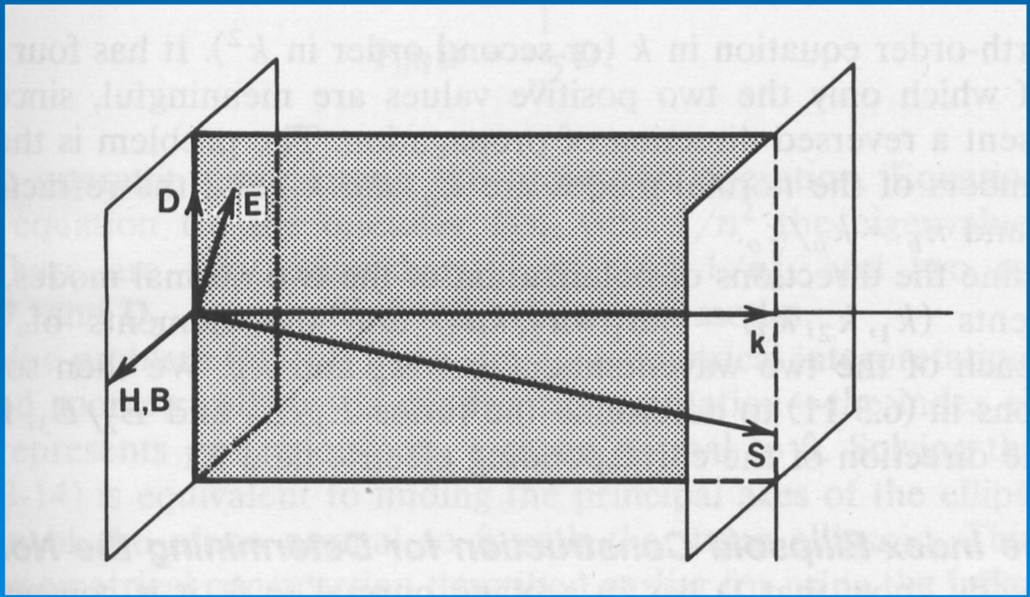


Electric field & Displacement Vector

$$\mathbf{k} \times \mathbf{H} = -\omega \mathbf{D}$$

$$\mathbf{k} \times \mathbf{E} = \omega \mu_0 \mathbf{H}$$





Summary

Linear optics: $D = \varepsilon_0 \overline{\overline{\varepsilon_r}} E \Longrightarrow P_i = \varepsilon_0 \chi_{ij} E_j$

Non-linear optics:

$$P^{NL}(\omega) = \varepsilon_0 \left[\chi^{(1)} E(\omega_1) + \chi^{(2)} E(\omega_1) E(\omega_2) + \chi^{(3)} E(\omega_1) E(\omega_2) E(\omega_3) + ...\right]$$

Linear electro-optic: $P^{NL}(\omega) = \varepsilon_0 \left[\chi^{(1)} + \chi^{(2)} E^{DC}\right] E(\omega_1)$

(Uniaxial crystal becomes biaxial when the field is applied)

$$\Delta n = |n_{x'} - n_{y'}| = n_0^3 r_{63} E^{DC}$$

$$V_{\lambda/2} = \lambda / (2 n_0^3 r_{63})$$