Remarks on identical particles in de Broglie–Bohm theory

Harvey R. Brown [a,1], Erik Sjöqvist [b,2], Guido Bacciagaluppi [a,3]

[a] *Sub-Faculty of Philosophy, Oxford University, 10 Merton Street, Oxford OX1 4JJ, UK*
[b] *Department of Quantum Chemistry, Uppsala University, Box 518, S-75120 Uppsala, Sweden*

Abstract

It is argued that the topological approach to the (anti-)symmetrisation condition for the quantum state of a collection of identical particles, defined in the "reduced" configuration space, is particularly natural from the perspective of de Broglie–Bohm pilot-wave theory. © 1999 Published by Elsevier Science B.V.

1. Introduction

Probably no detailed treatment of identical particles in non-relativistic quantum mechanics has been more influential than that due to Messiah and Greenberg [1]. At least as far as the strict consequences of the indistinguishable nature of the particles are concerned, these authors are led more to clarify constraints on the observables associated with a collection of such particles than constraints on the state of the collection. In particular, unlike Girardeau [2] and Mirman [3], for example, they do not appear to regard it to be a consequence of the indistinguishability of the particles that the configuration space wave function satisfies the condition

$$\psi(x_{P^{-1}1}, x_{P^{-1}2}, \ldots, x_{P^{-1}N})$$
$$= e^{i\gamma}\psi(x_1, x_2, \ldots, x_N), \tag{1}$$

where N is the number of particles, P is an arbitrary permutation on the set $\{1, 2, \ldots, N\}$, and γ is a real number which may or may not depend on the point in the configuration space. This seems, indeed, to be a separate assumption; after all, the square of the modulus of the wave function in (1) is not (contrary to Ref. [3, p. 113]) equal to the probability of detecting N particles in the spatial configuration $x_1, x_2, \ldots, x_N$, [4] the expression of which should be invariant under a permutation of the particle labels.

A separate and powerful approach to identical particles, however, does support the validity of (1), with the restriction that $e^{i\gamma}$ be a global phase factor. This is the approach based on the use of a reduced configuration space, in which configurations related by permutations are identified [5]. As early as 1927, Einstein [5] had voiced misgivings about Schrödinger's use of the

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[1] E-mail: harvey.brown@philosophy.oxford.ac.uk.
[2] E-mail: eriks@Kvac.uu.se.
[3] E-mail: guido.bacciagaluppi@philosophy.oxford.ac.uk.

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[4] This probability is given by $\sum |\psi(x_{P^{-1}1}, x_{P^{-1}2}, \ldots, x_{P^{-1}N})|^2$, where the sum is over all $N!$ permutations P; see Ref. [4, p. 584]. This is of course also the probability of finding any out of N *distinguishable* particles at the points $x_1, x_2, \ldots, x_N$, respectively.

[5] Notice that if the phase γ in (1) is global, that is independent of the point in configuration space, then the wave function ψ generally becomes multi-valued (see below), unless of course γ is an integer multiple of π.

full configuration space formed by the N-fold Cartesian product of three-dimensional Euclidean space as the domain of the wave function, on the grounds that for a system of N identical particles, there seemed to be a tension between considering configurations related by permutations as distinct and the recent results on particle statistics. Note, however, that if wave functions are defined on the reduced configuration space, that would seem to force symmetry on the wave functions when written as functions on the full configuration space, that is, γ in (1) should be identically zero, and only bosonic, not fermionic, statistics would be derivable.

Nevertheless, Einstein was right. Later researchers were to vindicate the use of the reduced configuration space in deriving the statistics of identical particles. In particular, it was in the profound analysis of Laidlaw and DeWitt [6] and particularly Leinaas and Myrheim [7] that the role of the non-trivial global topology (multiple connectedness) of the reduced configuration space was established. These insights are summarised in the next section. Suffice it to say here that in the topological approach the condition (1) above is valid, with $e^{i\gamma}$ a global phase factor, even if it is not quite a simple consequence of the indistinguishability of the particles. And remarkably, it can be shown that if the physical space has at least three dimensions, then the phase factor in (1) must have the values ± 1. In other words, the (anti-)symmetrisation condition on the wave function – the origin of (fermionic) bosonic statistics – is now seen to be related to the dimensionality of space, in contrast to the Messiah and Greenberg analysis wherein the (anti-)symmetrisation condition receives the status of a postulate.

Some implications of this topological approach to the treatment of identical particles within the framework of the de Broglie–Bohm "pilot-wave" formulation of quantum theory [8–13] have recently been studied [14]. The purpose of the present paper is principally to stress one point not emphasised in Ref. [14], namely that the multiple connectedness of the reduced configuration space, which seems somewhat ad hoc in the standard formulation of the topological approach, receives a natural justification within de Broglie–Bohm theory. Some brief, and hopefully pertinent remarks will also be made regarding the role of the full configuration space for

distinguishable particles in this theory. (A separate investigation of identical particles and their statistics from the point of view of de Broglie–Bohm theory will be the subject of a further publication.)

2. Topological theory of identical particles

Consider a physical system of N identical particles that move in a d-dimensional Euclidean [6] physical space $\mathbb{R}^d$. (To avoid unnecessary complications we assume the wave function is a product of a spin part and a spatial part, and that the spins are parallel, as in the main argument of Ref. [7].) In standard quantum mechanics the wave function of the system is defined, as in the case of distinguishable particles, on the full product configuration space $\mathbb{R}^{Nd} = \mathbb{R}^d \times \cdots \times \mathbb{R}^d$. Now the first occurrence of the claim that the (anti-)symmetrisation condition on the wave function associated with this system is related to the dimensionality d appeared, to the best of our knowledge, in the work of Girardeau [2]. This author assumed, as we have mentioned, the validity of (1) from the outset, which he regarded as a definitional property of identical particles. Having taken this step, Girardeau correctly allowed for the a priori possibility that the phase γ depends on the configuration point, and exploited this possibility to construct a consistent, if somewhat idealised, model of three particles with hard cores moving in one spatial dimension with non-trivial boundary conditions, in which the wave function fails to satisfy the (anti-)symmetrisation condition. In an interesting argument, Girardeau further showed that such a failure is generally impossible when motion is extended to three dimensions. (The limitations of this proof will be seen shortly.)

An arguably more convincing argument to essentially the same end starts with Einstein's claim above, that the full configuration space $\mathbb{R}^{Nd}$ contains, in the case of indistinguishable particles, redundant information. It would surely seem natural to consider instead the "reduced" space $\mathbb{R}^{Nd}/S_N$, which is the quotient of $\mathbb{R}^{Nd}$ obtained by the action of the symmetric group S_N (the group of permutations P above). Note that in tak-

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[6] For discussions of various non-Euclidean spaces in connection with the topological theory to be discussed below, see Refs. [15,16].

ing $\mathbb{R}^{Nd}/S_N$ as the appropriate configuration space for a quantum treatment of the system, one is effectively operating in an analogous fashion to the standard use of a reduced configuration (and hence phase) space in the classical solution to the so-called Gibbs paradox related to mixing of identical gases. In both cases, the identification of points related by a permutation of particle labels should perhaps not be considered as an inevitable consequence of the intrinsic nature of the particles, but rather as a reasonable step whose justification lies with the success of the theory built on it[7]. At any rate, the detailed procedure for applying quantum theory to $\mathbb{R}^{Nd}/S_N$, rather than the full configuration space $\mathbb{R}^{Nd}$, was demonstrated by Laidlaw and DeWitt [6] using Feynman formalism for $d = 3$, and Leinaas and Myrheim [7] using Schrödinger quantisation for arbitrary d.

Now an important first step in this theory is the removal from $\mathbb{R}^{Nd}/S_N$ of all points corresponding to two or more particles occupying the same spatial position at the same instant. The removal of such coincidence points, which are singular in $\mathbb{R}^{Nd}/S_N$, is sometimes justified by considering the particles to be "impenetrable", but of course impenetrability is not a direct consequence of indistinguishability (and does not seem to hold for bosons). For the moment we shall simply assume that the appropriate configuration space is indeed the multiply connected set $Q = \mathbb{R}^{Nd}/S_N - \Delta$, where Δ is the set of coincidence points[8].

In order to see the implications of the multiple connectedness of Q, it is convenient (though by no means

necessary) to use the Feynman path integral formalism. Feynman paths that connect two points x' and x in Q cannot in general be continuously deformed into each other without crossing a point in Δ. This means that paths in the Feynman propagator $K(x, t; x', t')$ divide into homotopy classes each of which consists of paths that are homotopically equivalent. Explicitly, denoting the homotopy classes by $[\alpha]$ we may write [6,18,19]

$$K(x, t; x', t') = \sum_{[\alpha]} \chi([\alpha]) K_{[\alpha]}(x, t; x', t'), \qquad (2)$$

with $\chi([\alpha])$ phase factors and $K_{[\alpha]}(x, t; x', t')$ the Feynman propagator formed by the paths in $[\alpha]$. It is precisely the possibility of having different $\chi([\alpha])$ for different $[\alpha]$ that physically distinguishes quantum mechanics on a multiply connected space (such as Q) from quantum mechanics on a simply connected space (such as $\mathbb{R}^{Nd}$). A well-known example of quantum mechanics on a multiply connected space is the Aharonov–Bohm effect [20] where the $\chi([\alpha])$ are given by $\exp(in_{[\alpha]}\Phi)$, with Φ the magnetic flux and $n_{[\alpha]}$ the winding number that characterises $[\alpha]$ around the flux line, which is the singular point. In the case of identical particles, the possible values of $\chi([\alpha])$ are determined by the topology of Q. In three dimensions ($d = 3$), Q can be shown to be doubly connected and therefore $\chi[\alpha] = \pm 1$, whereas in the two-dimensional case ($d = 2$), Q is infinitely connected and any $\chi[\alpha]$ value may occur.

Translating back into the language of wave functions on the full configuration space $\mathbb{R}^{Nd}$, the implication of the topological considerations above is that in three dimensions, the phase factor in (1) is necessarily ± 1 (corresponding to bosonic and fermionic behaviour), but that in the case of particles constrained to move in two dimensions, the (global) phase factor could be arbitrary (and the wave function is multivalued). (Equivalently, a "statistics field" is introduced to characterise the different types of identical particles, and the wave function is always single-valued – in fact symmetric.) Identical particles in $\mathbb{R}^2$ that obey intermediate statistics between the bosonic and fermionic cases are called anyons [21]. The theory of such particles has been successfully applied to the fractional quantum Hall effect (see Refs. [22–24]), and provides considerable justification for the

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[7] In a penetrating analysis of Gibbs' paradox, Hestenes [17] has stressed that appeal to the reduced phase space for the purposes of determining entropy by counting states is ultimately justified only in relation to the physical operations of mixing and filtering of "like" gases. Whether the use of the reduced configuration space $\mathbb{R}^{Nd}/S_N$ in the case of identical quantum particles carries more a priori justification than the analogous procedure in the case of classical gas particles is a moot point.

[8] The insistence on removing the singular points appears to be part of standard wisdom [18]. We just wish to note, however, that one can subdivide paths in different equivalence classes according to their winding numbers around the singularities and corresponding distinct values of the phase factor $e^{i\gamma}$, even retaining the singular points, if one sets the wave function zero (as a necessary condition for particles other than bosons) at the singular points, thus allowing for a discontinuous behaviour of the phase when paths are continuously deformed through a singular point. (The paths will be considered equivalent if amplitudes and phases separately are continuous under deformation.)

topological approach outlined above. Anyons were independently predicted using a quite different approach in Ref. [25]. This approach is not dynamical in the sense of the de Broglie–Bohm one sketched below, but it does lead directly to Q as a possible configuration space.

It may be of interest here briefly to compare these results with the analysis of Girardeau [2] mentioned at the beginning of this section. Girardeau attempted to show that generally the phase factor in (1) equals ± 1, and that it is only in the case of three dimensions ($d = 3$) that its value must be global, i.e. independent of the point in the full configuration space. In the case of particles constrained to move in two dimensions, this result is inconsistent with the existence of anyons, for which the phase factor in question is global, being a property of the kind of particles involved, but not equal to ± 1. However, in his proof, Girardeau assumed the availability of real-valued wave functions, and it can be shown in the topological approach that systems of anyons must have complex-valued wave functions due to the broken time reversal symmetry associated with statistics phase factors different from ± 1 (see Ref. [19, pp. 134–135]).

3. De Broglie–Bohm theory

Let us return to the issue at the heart of the topological approach which is that of the removal of the set Δ of coincidence points from the reduced configuration space $\mathbb{R}^{Nd}/S_N$, rendering Q multiply connected. In the theory of anyons, it has been conjectured [26,27] that short-range repulsive forces are at work between the particles; the adoption of such a view in the general case of identical particles appears however ad hoc. (Note that within the topological approach it is only strictly necessary for non-bosonic behaviour.) We now argue that such ad hocness is removed in the framework of de Broglie–Bohm pilot-wave theory.

Recall that the de Broglie–Bohm formulation of non-relativistic quantum mechanics [8–13] posits, besides the wave function on $\mathbb{R}^{Nd}$ for an isolated system of N particles (in the case of such a system being in a pure state), a collection of N point corpuscles. It is the role of the wave function ("pilot-wave"), itself a solution of the time-dependent Schrödinger equation, to determine the instantaneous velocities of the cor-

puscles through the guidance equations

$$m_k \dot{X}_k = \nabla_{x_k} S|_{x=X}, \quad k = 1, \ldots, N, \quad (3)$$

where $\dot{X}_k$ is the velocity of the kth corpuscle with mass m_k, $S = S(x, t)$ is the phase of the pilot-wave (in units of $\hbar$) at the configuration point $x = (x_1, \ldots, x_N)$ and $X = (X_1, \ldots, X_N)$ is the instantaneous configuration of the N corpuscles. (Note that in the case of an external field acting on the system and which is represented by a vector potential, the right-hand side of (3) will incorporate an additive term linear in the vector potential, which in particular makes (3) gauge-invariant.)

A feature of pilot-wave theory that is important for our purposes is this. Whereas in standard quantum mechanics the points of the configuration space are interpreted as the possible results of position measurements or detections of the N particles, which are generally non-localised in $\mathbb{R}^d$ prior to the measurements, in de Broglie–Bohm theory they are normally interpreted as the possible positions of the N de Broglie–Bohm corpuscles. That is to say, they are truly objective configurations (as in classical theory), and make no reference to measurement or detection processes.

In a recent analysis [14] it has been demonstrated that the reduced configuration space approach is a natural framework for identical particles in de Broglie–Bohm theory. The identity of the particles implies that any two distinct initial configurations of the corpuscles that differ only by a permutation yield the same set of corpuscle trajectories $X_k(t)$, $k = 1, \ldots, N$, in $\mathbb{R}^d$. Again this shows that the full product configuration space $\mathbb{R}^{Nd}$ contains redundant information. Indeed, taking the restriction $\mathbb{R}^{Nd}/S_N$ as the configuration space, we obtain the same set of trajectories $\{X_k(t)\}$ in the physical space $\mathbb{R}^d$. The physical configuration space for a system of N identical particles in de Broglie–Bohm theory is therefore given by $\mathbb{R}^{Nd}/S_N$.

What is the status of the singular points with respect to the de Broglie–Bohm trajectories? Consider for simplicity the case of $N = 2$, and suppose that the two corpuscles have distinct positions in $\mathbb{R}^d$ at time t_0 and that they coincide at some finite time $t > t_0$. The single trajectory in $\mathbb{R}^{2d}/S_2$ which contains the coincidence point at t will generate two trajectories in the full configuration space, the point in one at any instant being obtained from that in the other "mirror" trajectory

by a permutation of particle labels. But such trajectories in $\mathbb{R}^{2d}$ will cross at the coincidence point, a possibility that is ruled out by the first order nature of the guidance equations (3). It follows that the coincidence points are inaccessible from non-coincidence configurations at t_0. Conversely, the time-reversal symmetry of the de Broglie–Bohm trajectories implies that two identical corpuscles that start at the same point in space (as two bosonic corpuscles could) will remain coincident forever. Intuitively, since these initial conditions (pilot-wave and spatial position) are entirely symmetric with respect to the two corpuscles, and we are assuming that the symmetry of the pilot-wave is preserved over time, we do not expect their future or past trajectories to differ, and so the corpuscles coincide forever, if at all.

If need be, we can put this argument on a more rigorous footing. Writing $\psi = Re^{iS/\hbar}$ for the system in centre of mass coordinates $x = x_1 - x_2$ and $x_{CM} = \frac{1}{2}(x_1 + x_2)$, it follows from the symmetry or antisymmetry (or given the anyonic phase relation) of ψ that $S(-x) = S(x) + \gamma$, which implies that $\nabla_x S(x) = 0$ at $x = 0$. We then obtain immediately from the relation $\nabla_x = \frac{1}{2}(\nabla_{x_1} - \nabla_{x_2})$ that $\nabla_x S = m\frac{1}{2}(\dot{X}_1 - \dot{X}_2) = 0$ at $x = 0$. From the vanishing relative velocity at the coincidence points and the first-order nature of the guidance equation (3) one concludes that two initially separated corpuscles for identical particles cannot reach the same point in the physical space at the same time.

The generalisation of this conclusion to the case of arbitrary N is straightforward. Thus, the sets corresponding to the M-point coincidences for any $M \leqslant N$, with their union Δ, as well as the set $Q = \mathbb{R}^{Nd}/S_N - \Delta$, are invariant submanifolds of the reduced configuration space $\mathbb{R}^{Nd}/S_N$ under the action of the de Broglie–Bohm dynamics (3). In particular, the space Q of regular points can be used consistently as a configuration space for a de Broglie–Bohm theory. Further, removal of the sets of M-point coincidences seems physically well motivated, since they correspond to motions for which M particles coincide for all times – which would appear as the motion of one particle of M-fold mass and charge. (And removal of these sets does not affect the statistical predictions of de Broglie–Bohm theory, since they have total $|\psi|^2$-measure zero.) We thus argue that within the topological approach to identical particles the removal of the set Δ of coincidence points from the reduced configuration space $\mathbb{R}^{Nd}/S_N$ thus

follows naturally from de Broglie–Bohm dynamics as it is defined in the full space $\mathbb{R}^{Nd}$.

We finish this section by removing a possible source of confusion. It is well known that the nodal set (that is, the set of zeros) of the wavefunction can be shown to be accessible from the outside at most for a set of initial conditions of $|\psi|^2$-measure zero (see Ref. [28]). It is also well known that the wave function of a system of bosons (unlike that of a system of fermions) need not vanish at a coincidence point in $\mathbb{R}^{Nd}$. As a consequence it has been claimed that the de Broglie–Bohm trajectories of bosonic corpuscles may cross in the physical space (see Ref. [10, p. 284]). But we have just seen how de Broglie–Bohm dynamics secures the inaccessibility of singular points in $\mathbb{R}^{Nd}/S_N$, or coincidences in $\mathbb{R}^{Nd}$, without qualifications and irrespective of whether they also correspond to nodal points. So, if the above claim is meant in the sense that particles starting from different points in space can cross, it is contradicted by our above result. If on the contrary it means merely that de Broglie–Bohm theory in the full configuration space is able to describe coincident bosons, then it is obviously correct. We have only suggested that it would be more natural to exclude such coincidences, because we have shown they would hold forever. The difference between the case of bosons and that of fermions lies not in a non-zero probability for coincidence of bosons (given that the set of coincidences has $|\psi|^2$-measure zero irrespective of whether or not the wave function vanishes at the coincidences); it lies rather in the fact that while at a node the phase of the wave function is ill-defined, and thus the de Broglie–Bohm dynamics breaks down for coincident fermions, the trajectories of coincident bosons, if one should wish to retain them, would be well-defined for all times.

4. Distinguishable particles

One can easily convince oneself of the fact that the first-order nature of the guidance equations (3) implies that the guidance equations will continue to be first order when the wave function of the system of identical particles is defined on $\mathbb{R}^{Nd}/S_N$, so that for any point in $\mathbb{R}^{Nd}/S_N$ there will be a single curve in this space containing it. This orbit gives rise to $N!$ curves in $\mathbb{R}^{Nd}$, each related to any other by a permutation

of particles labels. But suppose the particles are distinguishable; then the guidance equations tell us that curves in $\mathbb{R}^{Nd}$ that at a given instant contain points related by such permutations generally do not continue to be thus related at other times, and so cannot be generated from a curve in $\mathbb{R}^{Nd}/S_N$.

The fact that the correct quantum mechanical treatment of a system of N distinguishable particles requires the wave function to be defined on $\mathbb{R}^{Nd}$, rather than $\mathbb{R}^{Nd}/S_N$, suggests strongly that the hypothetical corpuscles in de Broglie–Bohm theory associated with such a system are pairwise distinct, or "labelled". After all, given both the meaning of the configuration space in this theory (see above) and the apparent success of the topological approach for identical particles, it would be awkward (but perhaps not inconsistent – see Footnote 7) to maintain that the point corpuscles were intrinsically identical, apart from their spatial positions, while assuming that the correct domain of the pilot-wave for the system is $\mathbb{R}^{Nd}$.

What properties possessed by the de Broglie–Bohm corpuscles in this case serve to label them? The obvious answer seems to be that it is the same properties that distinguish the particles in the conventional theory, viz. mass, charge, magnetic moment etc. Indeed what else could they be? But it has not escaped notice that certain interference experiments involving single particles seem to suggest that these dynamical properties pertain to the pilot-wave[9]. This does not mean that they cannot also belong to the corpuscle. Arguments in favour of this "principle of generosity" (related, e.g., to the "inertia" of the corpuscles) in de Broglie–Bohm theory – the assignment of such properties as mass and charge to both the pilot-wave and the corpuscle – have indeed been given in the literature (for a review see Ref. [30]). The dynamical considerations raised in the previous paragraphs provide in our opinion another argument in favour of the principle of generosity.

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[9] See Refs. [29,30]. As an example involving charge [29], consider the Aharonov–Bohm effect [20] referred to in the previous section. The expression for the phase shift due to the flux in the shielded solenoid depends on charge being present on spatial loops within the support of the wave function and enclosing the solenoid, whereas the trajectory of the de Broglie–Bohm corpuscle associated with the charged particle does not encircle the solenoid (see Ref. [11, Section 3.8]).

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