University of Calgary
Winter semester 2017

PHYS 543: Quantum Mechanics II

Third midterm examination

November 15, 2017, 10:00–10:50, ST 061

Problem 1. Find the Clebsch-Gordon coefficient

a) $\langle l_1 = 5, m_1 = 5, l_2 = 3, m_2 = 3 |\, l_1 = 5, l_2 = 3, l = 8, m = 8 \rangle$;

b) $\langle l_1 = 5, m_1 = 5, l_2 = 3, m_2 = 2 |\, l_1 = 5, l_2 = 3, l = 8, m = 7 \rangle$.

Problem 2. Alice and Bob share a pair of photons in the polarization state $|\Psi\rangle = \frac{1}{3}(|HH\rangle + 2\,|HV\rangle - 2\,|VV\rangle)$.

a) Alice measures her photon in the diagonal basis. For each possible measurement result, find the state in which Bob's photon will be prepared and the corresponding probability.

b) From the answer in part (a), calculate the density matrix of Bob's photon in the canonical basis, assuming that Bob does not know Alice's result.

c) Calculate Bob's reduced density matrix using partial trace. Check consistency with part (b).

Problem 3. In a spin echo experiment, instead of the standard excitation pulse sequence $\left(\frac{\pi}{2}, \pi\right)$ applied at $t = (0, \tau)$, the sequence (θ, π) is used. Find the behavior of the echo signal amplitude at $t = 2\tau$ as a function of θ (the absolute magnitude of the signal need not be evaluated).
Hint: Solving this problem requires no calculations. Visualizing the behavior of the Bloch vector is sufficient to find the answer.