PHYS 543 Fall 2017

Second midterm

Solutions

1 $L^2 |\ell\, e\rangle = \hbar^2 \ell(\ell+1) |\ell\, e\rangle$

$$\Rightarrow \langle \ell\, m | L^2 | \ell\, e\rangle = \hbar^2 \ell(\ell+1)\delta_{em}$$

$$L_z |\ell\, e\rangle = \hbar m |\ell\, e_m\rangle$$

$$\Rightarrow \langle \ell\, m | L_z | \ell\, e\rangle = \hbar e \delta_{em}$$

$$L_x |\ell\, e\rangle = \frac{L_+ + L_-}{2} |\ell\, e\rangle = \frac{\hbar}{2}\sqrt{\ell(\ell+1) - e(e-1)}\, |\ell, e-1\rangle$$

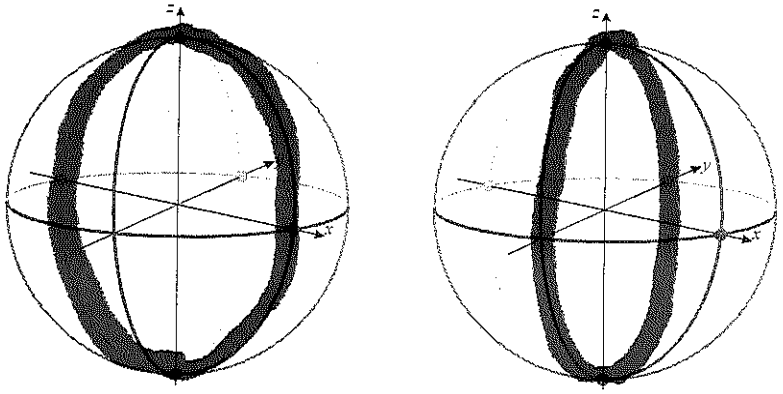
$$= \hbar\sqrt{\frac{e}{2}}\, |\ell, e-1\rangle$$

$$\Rightarrow \langle \ell\, m | L_x | \ell\, e\rangle = \hbar\sqrt{\frac{e}{2}}\,\delta_{e-1,m}$$

2 a) $|\alpha\rangle = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \Rightarrow \theta = 2\alpha,\ \varphi = 0 \text{ or } \pi$

b) $\langle \alpha | \Psi^- \rangle = \frac{1}{\sqrt{2}}(\cos\alpha |H\rangle + i \sin\alpha |V\rangle) \rightarrow \begin{pmatrix} \cos\alpha \\ i\sin\alpha \end{pmatrix}$

$$\Rightarrow \theta = 2\alpha,\ \varphi = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$



3 $\psi(r,\theta,\varphi) = R_{n,n-1}(r)\, Y_{n-1}^{n-1}(\theta,\varphi)$

$R_{n,n-1}(r) \propto r^{n-1} e^{-r/na}$ (Ex. 4.43)

$Y_{n-1}^{n-1}(\theta,\varphi) \propto \sin^{n-1}\theta\, e^{i(n-1)\varphi}$.