University of Calgary
Winter semester 2017

PHYS 543: Quantum Mechanics II

First midterm examination

October 4, 2017, 10:00–10:50, ST 061

Problem 1. (20 points) A coherent state $|\alpha\rangle$ with $\alpha = 2i$ is subjected to one of the following transformations:

a) position displacement by $X_0 = 3$;

b) phase shift $F(\phi) = e^{i\phi\hat{n}}$ with $\phi = \pi/4$;

c) squeezing by the factor $e^{-r} = \frac{1}{2}$ of the position observable.

Show the initial and resulting states by "uncertainty circles" in the phase space. Write the mean values and uncertainties of their position and momentum. You are not required to show the calculation of your answer.

	$\langle X \rangle$	$\langle P \rangle$	$\langle \Delta X^2 \rangle$	$\langle \Delta P^2 \rangle$
original				
displaced				
phase shifted				
squeezed				

Problem 2. (30 points) Find the commutator $[[\hat{L}_j, \hat{r}_k], [\hat{L}_m, \hat{p}_n]]$ for arbitrary j, k, m, n.

Problem 3. (50 points) A particle, initially in the vacuum state, is evolving under the Hamiltonian

$$\hat{H} = \hbar\gamma\hat{P}^2.$$

a) Find the evolution of the position and momentum observables as a function of time t in the Heisenberg picture.

b) Find the mean and variance of the observable $\hat{Y} = \frac{X-P}{\sqrt{2}}$ as a function of time. At which t does this observable exhibit squeezing, i.e. $\langle \Delta Y(t)^2 \rangle < \langle \Delta Y(0)^2 \rangle$?