**University of Calgary**
**Fall semester 2015**

## PHYS 543: Quantum Mechanics II

# Midterm examination 1

November 18, 2015, 10:00–10:50

Open books. No electronic equipment allowed.
Full credit = 100 points. Attempt all problems. Partial credit will be given.

**Problem 1.** Two photons, associated with fictitious observers Alice and Bob, are in state

$$|\Psi\rangle = a\,|{+}{+}\rangle + b\,|{-}{-}\rangle\,,$$

where $a$ and $b$ are real and non-negative; $a > b$. Alice's photon is lost.

a) What condition must $a$ and $b$ satisfy in order for $|\Psi\rangle$ to be physical (5 pts)?

b) Find the density operator of Bob's photon in the diagonal and canonical bases (25 pts).

c) Find the purity of the state of Bob's photon (10 pts). Find the values of $a$ and $b$ for which the state is pure and fully mixed.

d) Find the Cartesian and spherical coordinates of the corresponding Bloch vector (15 pts).

**Problem 2.** An electron in an atom with orbital quantum number $l$ experiences spin-orbit interaction with Hamiltonian $\hat{H} = H_0 + A\hat{\vec{L}} \cdot \hat{\vec{S}}$, where $H_0$ and $A > 0$ are known constants, $\hat{\vec{L}}$ and $\hat{\vec{S}}$ are the electron's orbital and spin angular momenta. In addition, a magnetic field $\vec{B}$ is applied.

a) Find the energy eignevalues in the absence of the field (15 pts).

b) Into how many sublevels will each of these levels split in the presence of a weak magnetic field (5 pts)?

c) When the magnetic field is strong, the dependence of $E(B)$ of the energy levels on the magnetic field is close to linear. Find all possible asymptotic values of the slopes $\mathrm{d}E/\mathrm{d}B$. How many energy eigenstates are associated with each slope (25 pts)?