University of Calgary
Fall semester 2015

PHYS 543: Quantum Mechanics II

Midterm examination 1

October 21, 2015, 10:00-10:50

Open books. No electronic equipment allowed.
Full credit = 100 points. Attempt all problems. Partial credit will be given.

Problem 1. Let $\hat{L}^y_\pm = \hat{L}_z \pm i\hat{L}_x$. Find

a) $[\hat{L}^y_+, \hat{L}^y_-]$ (15 pts);

b) $[\hat{L}^y_+, \hat{y}]$ (15 pts).

Problem 2. The electron in a hydrogen atom is prepared in a state that is a simultaneous eigenstate of the following observables:

- energy with eigenvalue $\sim -(13.6/4)$ eV,
- orbital angular momentum squared with eigenvalue $2\hbar^2$,
- projection of the orbital angular momentum onto the z axis with eigenvalue 0.

a) Write the wavefunction of that state in the position basis, circular coordinates (10 pts).

b) Write the complete spanning set of states of the electron with this energy in the form of quantum numbers $|n, l, m, m_s\rangle$ (10 pts).

Problem 3. An electron spin, initially in the $|\uparrow\rangle$ state, is placed into magnetic field $\vec{B}$ of a known magnitude oriented in the x–z plane at a 45° angle between the positive x and z axes.

a) Write the Hamiltonian in the form of a 2×2 matrix in the canonical basis (15 pts).

b) How long will the evolution take until the spin is pointing in the positive x direction (15 pts)?

c) Suppose the evolution described above is occurring in the rotating basis under a magnetic resonance setting, so $\vec{B}$ is the fictitious field. The actual dc field B_0 is applied along the z axis, rf field $B_{\rm rf} \cos \omega t$ along the x axis. Find $B_{\rm rf}$ (20 pts).