University of Calgary
Winter semester 2011

PHYS 443: Quantum Mechanics I

Homework assignment 5

Due March 29, 2011

Problem 5.1. Find the Fourier transform of the following functions (with $\kappa, a > 0$).

a) $f(x) = \delta(x+a) + \delta(x-a)$.

b) $f(x) = \cos \kappa x$.

c) $f(x) = \sin \kappa x$.

d) $f(x) = e^{-\kappa|x|}$.

e) $f(x) = \begin{cases} 1 & \text{if } -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$

f) $f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 2a \\ 0 & \text{otherwise} \end{cases}$. Calculate your answer (i) by direct integration and (ii) from part (e) using the expression (C.23) for the Fourier transform of a shifted function. Verify that the two answers are consistent.

g) $f(x) = f(x) = e^{-\kappa|x|} \cos x$. Calculate your answer (i) by direct integration and (ii) noting that $f(x)$ is the product of the functions from parts (b) and (d) and using the fact that the Fourier transform of a product is a convolution. Verify that the two answers are consistent.

Problem 5.2.

a) Show that

$$e^{-i\hat{p}a/\hbar} |x\rangle = |x+a\rangle$$

b) Express the wavefunction of state $e^{-i\hat{p}a/\hbar} |\psi\rangle$ through the wavefunction $\psi(x)$ of state $|\psi\rangle$.

c) Express wavefunctions $\tilde{\psi}(p) = Ae^{-b^2p^2/2}$ (where A is the normalization factor) and $\tilde{\psi}'(p) = e^{-i\hat{p}a/\hbar}\tilde{\psi}(p)$ in the position basis and verify consistency with part (b).

Problem 5.3. A state has wavefunction

$$\psi(x) = Axe^{-k^2x^2/2}.$$

a) Find the normalization factor A.

b) Find the wavefunction $\tilde{\psi}(p)$ in the momentum basis.

c) Verify the uncertainty principle.

Hint: the expression for the Fourier transform of a derivative may be useful.

$$\int_{-\infty}^{+\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{2}; \quad \int_{-\infty}^{+\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{4}.$$