UNIVERSITY OF CALGARY

Physics 443: Quantum Mechanics I
First midterm examination

February 15, 2005

1. Consider an apparatus for measuring photon polarization that has the following properties:

- whenever it is fed with a +30° polarization photon, it displays "2"
- whenever it is fed with a +120° polarization photon, it displays "3"
- for photons with polarizations other than the above, it randomly displays one of these numbers with some probabilities

a) [5 pts] Write the operator $\hat{A}$ associated with the observable measured by this apparatus in the Dirac notation

b) [10 pts] Find the matrices of $\hat{A}$ in its eigenbasis and in the $\{|H\rangle, |V\rangle\}$ - basis

c) [5 pts] Find the probability of each measurement outcome for a horizontally polarized photon input state

d) [20 pts] Find the expectation value and the mean square uncertainty of the observable $\hat{A}$ for a horizontally polarized input.

Note: $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

2. An operator $\hat{A}$ has the following matrix in the $\{|H\rangle, |V\rangle\}$ - basis: $\begin{pmatrix} -3 & 4i \\ -4i & 3 \end{pmatrix}$

a) [3 pts] Verify that $\hat{A}$ is Hermitian

b) [20 pts] Find its eigenvalues $\{a_1, a_2\}$ and corresponding eigenvectors $\{|a_1\rangle, |a_2\rangle\}$ in the $\{|H\rangle, |V\rangle\}$ - basis

c) [7 pts] Verify that the set $\{|a_1\rangle, |a_2\rangle\}$ forms an orthonormal basis

d) [10 pts] Verify explicitly that $\hat{A} = a_1 |a_1\rangle\langle a_1| + a_2 |a_2\rangle\langle a_2|$

e) [20 pts] At the moment $t = 0$ the state of the photon was measured to be $|H\rangle$. What is the probability to find the photon in the state $|+45^\circ\rangle$ at the moment $\omega t = \pi / 10$ assuming it evolves under the Hamiltonian $\hat{H} = \hbar\omega\hat{A}$?