Four lectures on basic applications of statistical mechanics.

Suitable lecture notes for this part of the course: Prof J. Devriendt

- https://www.physics.ox.ac.uk/our-people/devriendt/teaching

sections 8,9,10.

The graphs and illustrations in these lectures are mostly taken from these notes (with permission).

Timing of lectures

This part: 4 lectures, Wed, Thur, Fri 12:00 this week

AND Wed 11:00 next week (4th week)

then Prof Schekochihin continues from 12:00 that day.

I return with thermodynamics in 6th week, from Wednesday, with a further 7 lectures, finishing in 8th week.

This lecture

1. Review some basic results/concepts

2. Take a detailed look at microstates, macrostates, occupation numbers and counting microstates.

3. Look at two-state system, e.g. spin-half paramagnetic solid.

Example

A set of N = 20 distinguishable harmonic oscillators,

with total energy of system E = 10 units (above ground state).

Total number of microstates W = 20,030,010

Number of microstates in most likely $\{n_i\}$ is w = 3,527,160

W/w = 5.68, ln(W) = 16.8 and ln(w) = 15.1 (16.8/15.1 = 1.11)

If N = 40 oscillators, and E = 20 then W = 2794560862883130

w = 232301367698400

W/w = 12, ln(W) = 35.6, ln(w) = 33.1 (35.6/33.1 = 1.07)

The formula to be derived in next lecture gives 20 ln(27/4) = 38.2

The magic of logarithms and really huge numbers

For a system with entropy $S = N_A k_B$ we have

$W = \exp(N_A)$

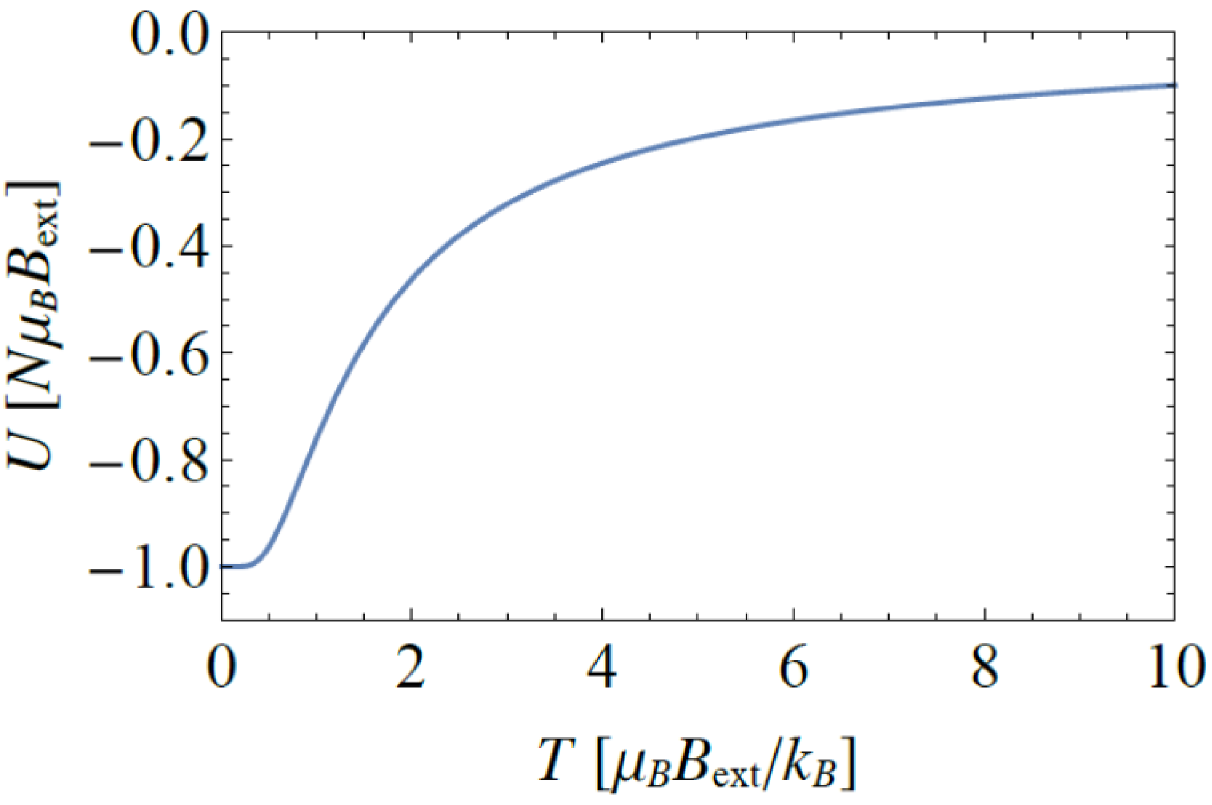
Now suppose we estimated W incorrectly by a large factor such as 1 billion. How bad is that?

$$S_{estimate} = k_B \ln(10^9 \exp(N_A)) = k_B \, (9 \ln(10) + N_A)$$

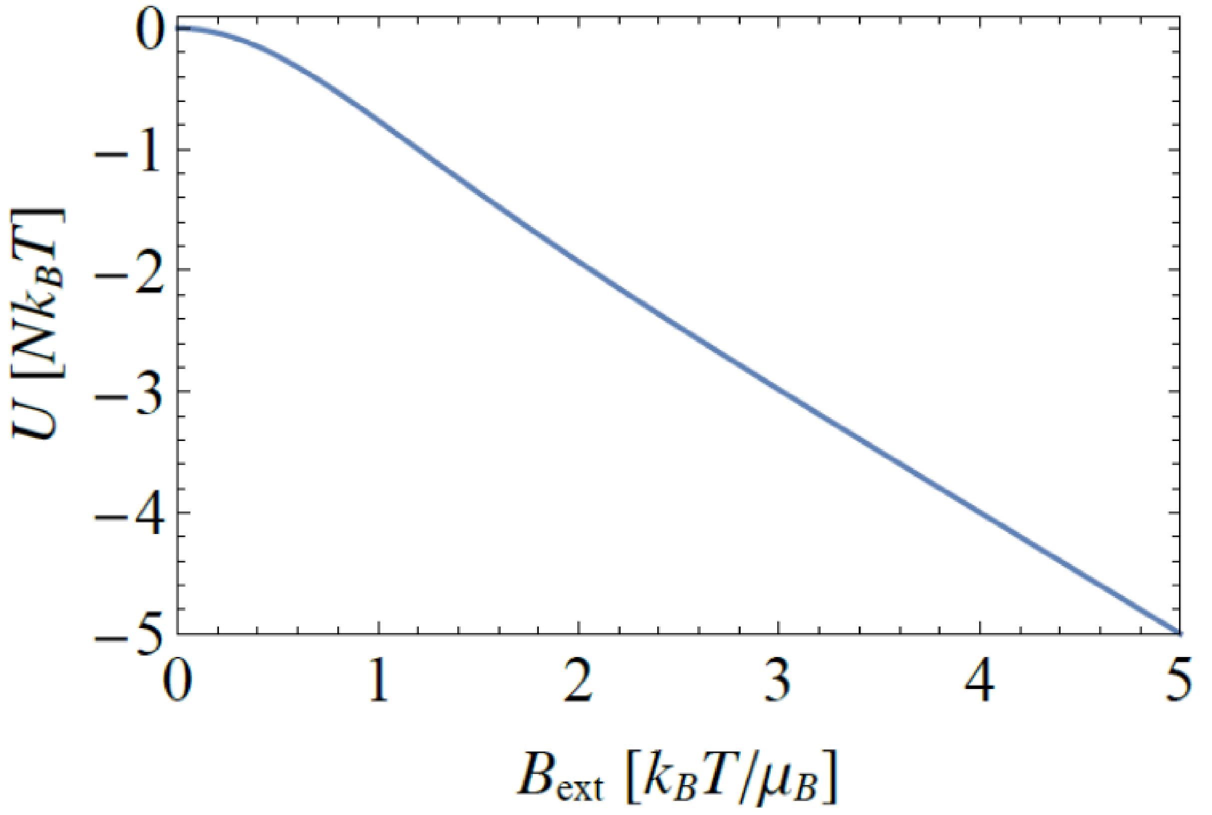
Hence the fractional error in our entropy estimate

would be $\frac{9 \ln(10)}{N_A} = 3 \times 10^{-23}$

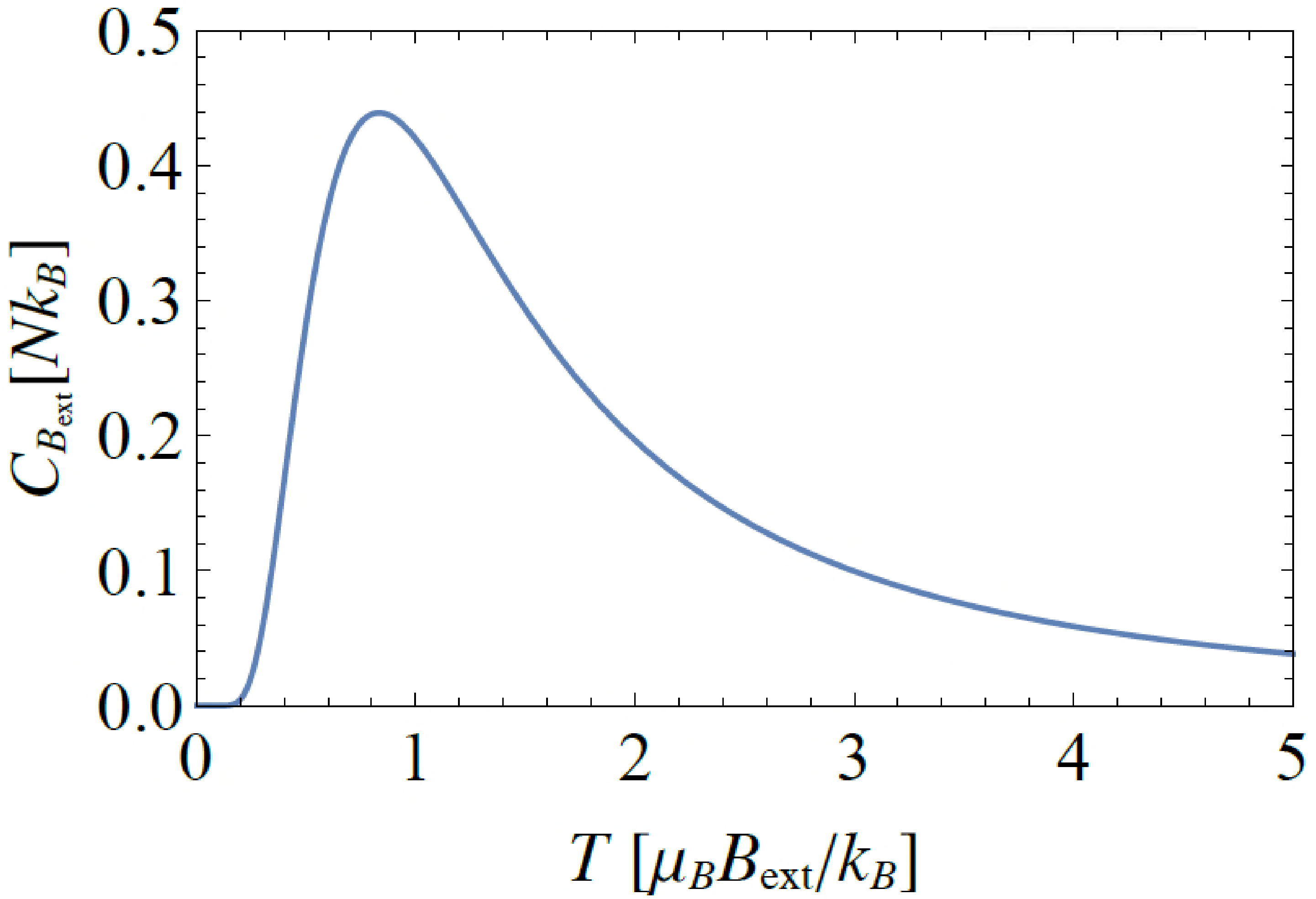
(Notice that W is a really huge number here.)



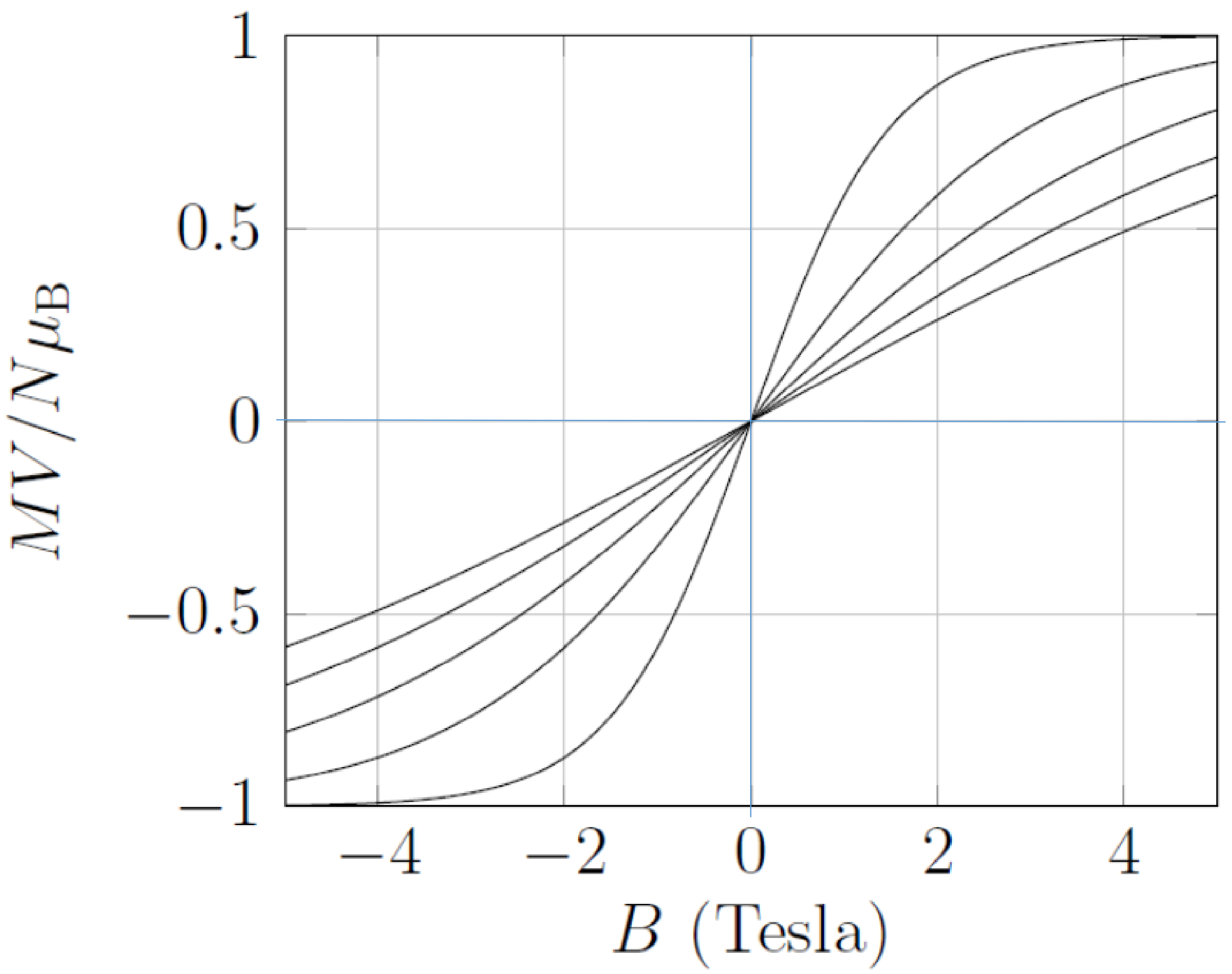
Internal energy of a spin-half paramagnet as a function of temperature.



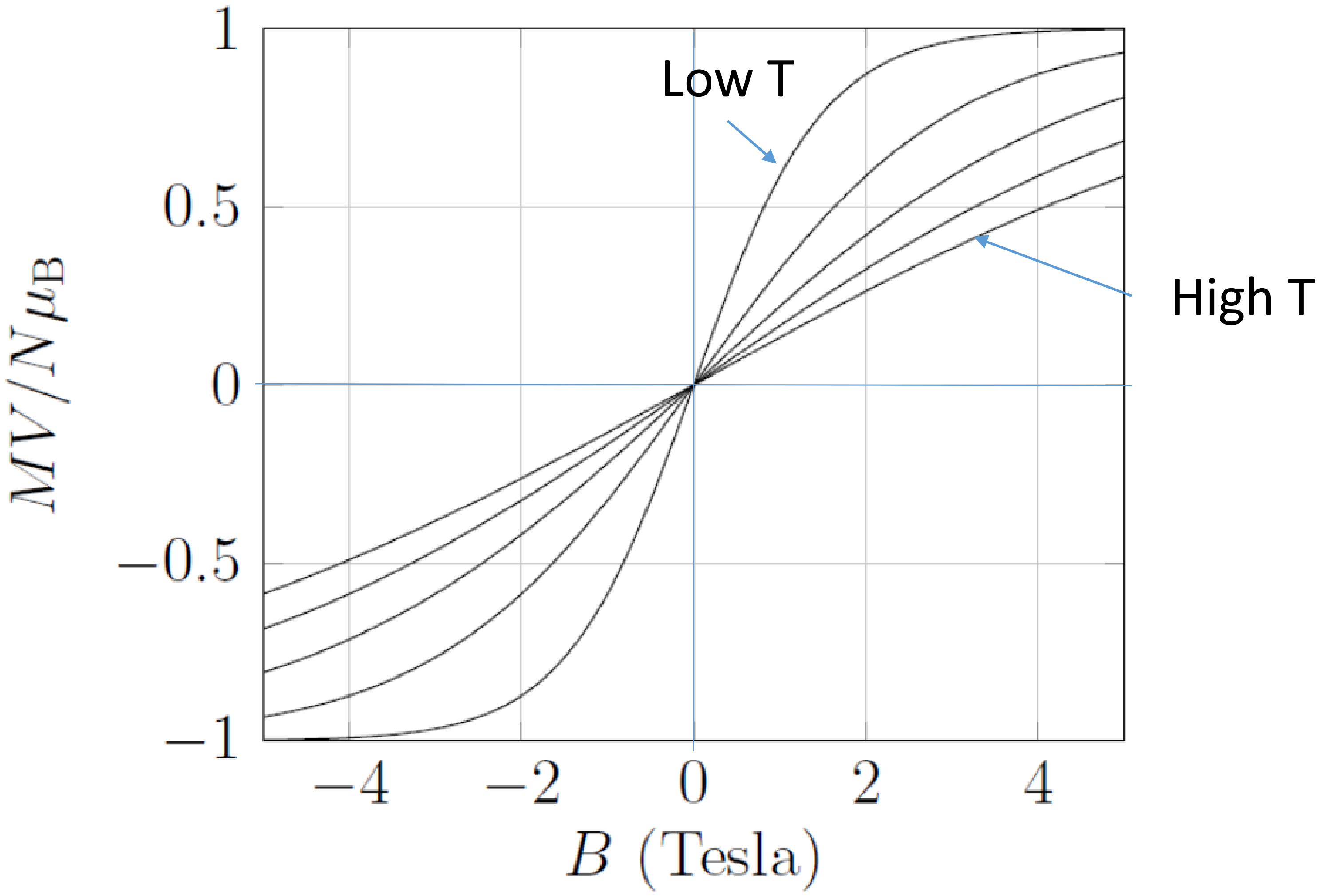
Internal energy of a spin-half paramagnet as a function of applied magnetic field.



Heat capacity of a spin-half paramagnet as a function of temperature.
(This functional form or shape is called a “Schottky anomaly”)



Magnetization as a function of applied magnetic field, for temperatures 1 to 5 kelvin.



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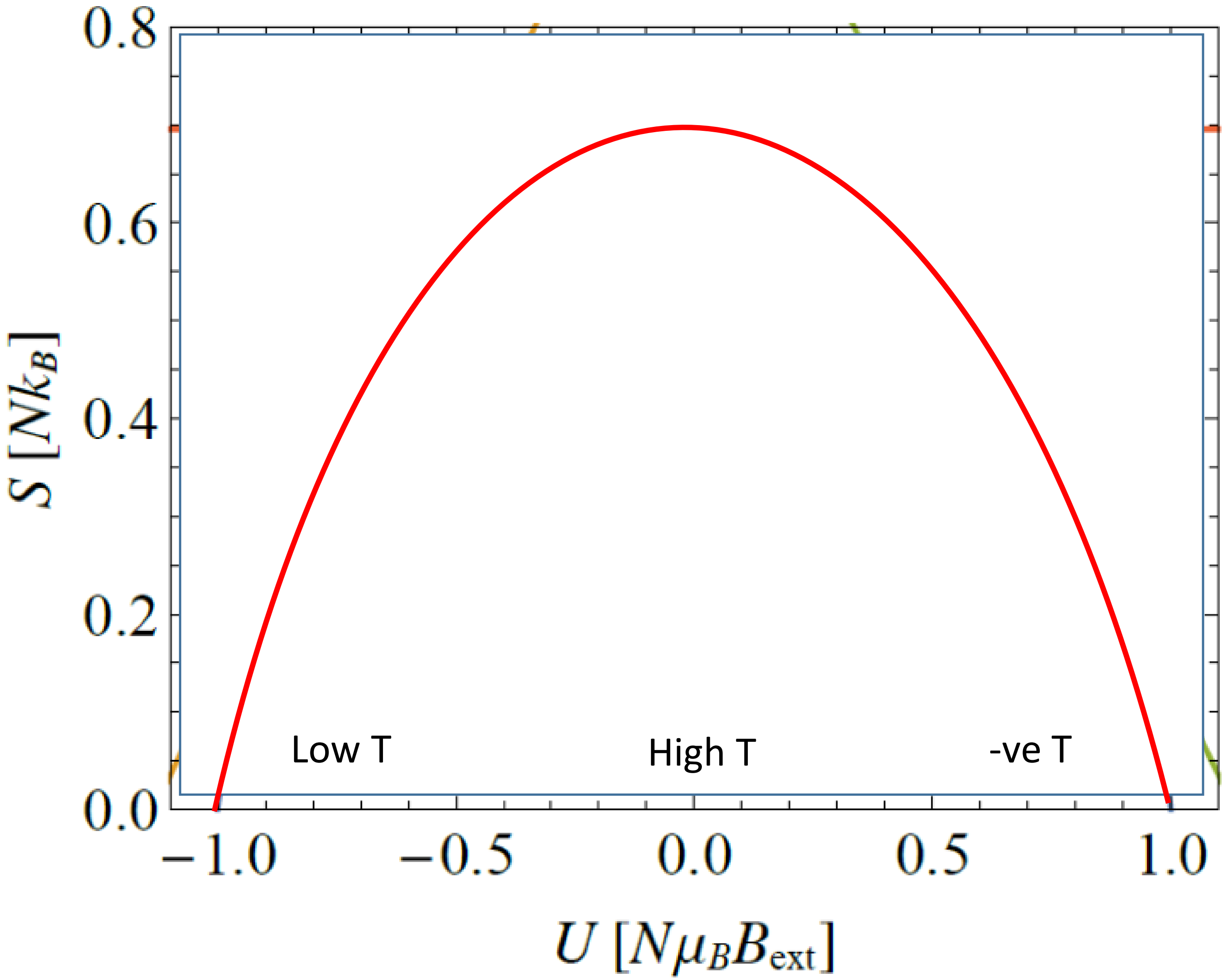
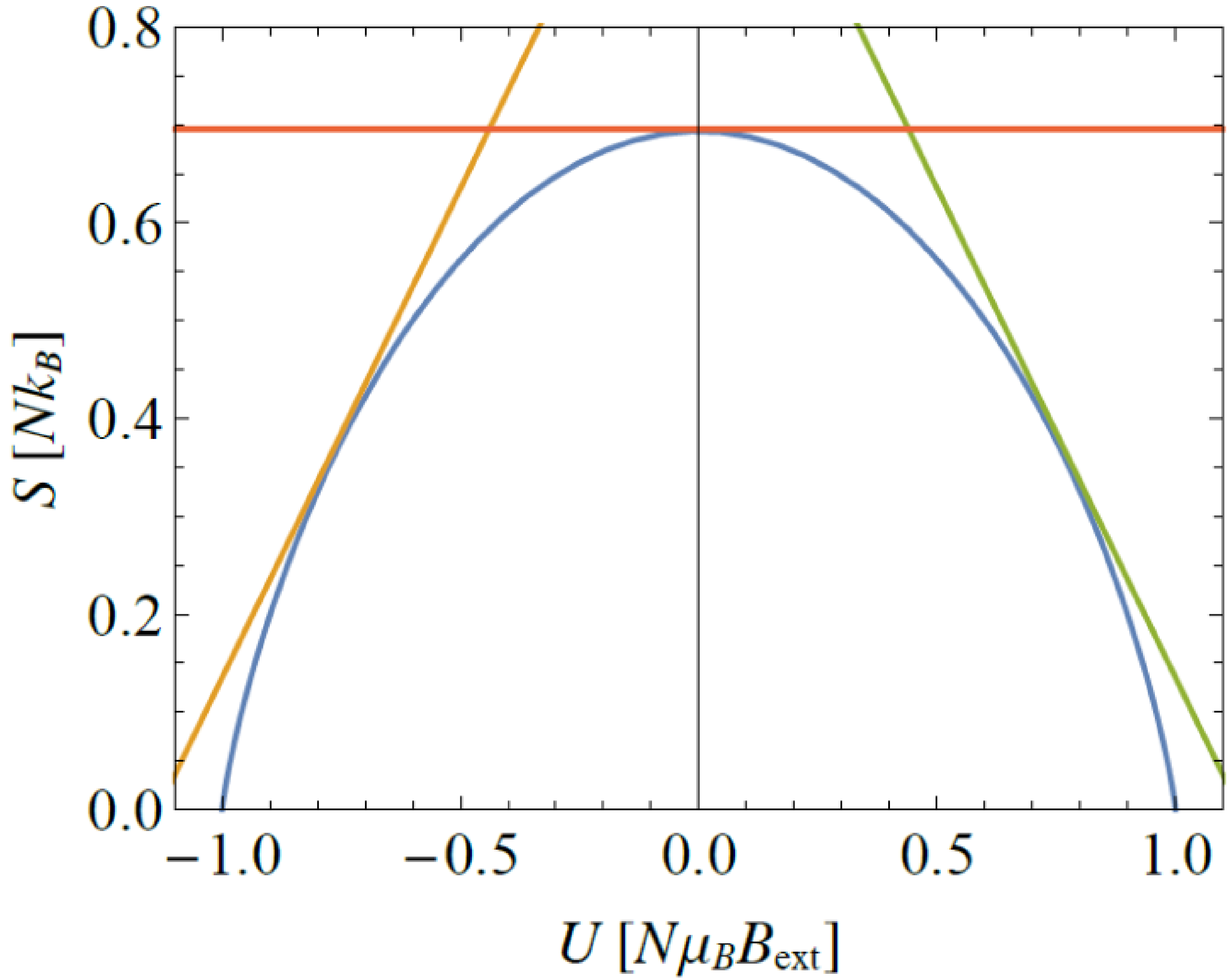


Entropy of a paramagnet as a function of internal energy.



Entropy of a paramagnet as a function of internal energy.

The straight lines give three examples of the slope (dS/dU).